RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4771

Decision Mathematics 1

Friday **14 JANUARY 2005** Morning 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Questions **2**, **4** and **5**.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 7 printed pages, 1 blank page and an insert.

 [Turn over

Section A

1 The bipartite graph in Fig. 1 represents a board game for two players. At each turn a player tosses a coin and moves their counter. The graph shows which square the counter is moved to if the coin shows heads, and which square if it shows tails. Each player starts with their counter on square 1. Play continues until one player gets their counter to square 9 and wins.

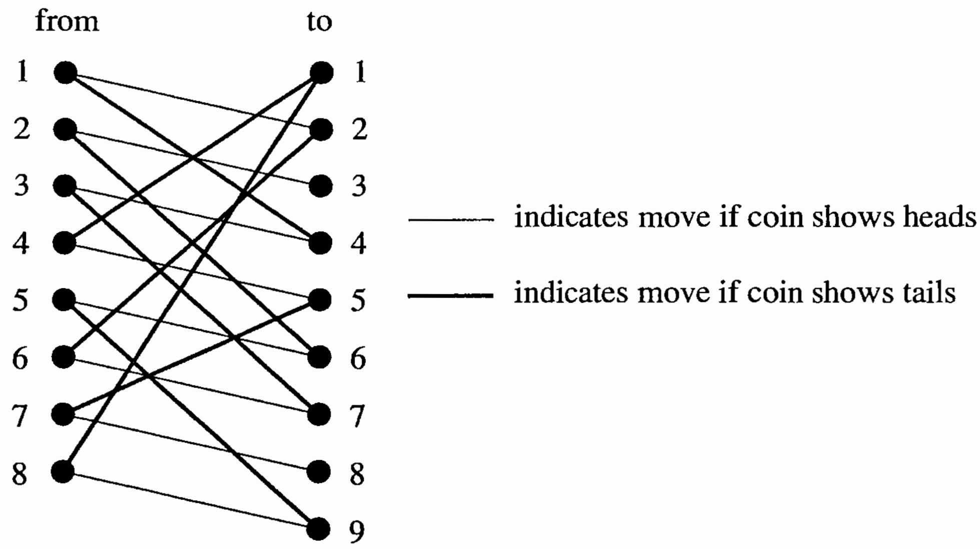

Fig. 1

(i) Draw a tree to show all of the possibilities for the player's first three moves. [4]

(ii) Show how a player can win in 3 turns. [1]

(iii) List all squares which it is possible for a counter to occupy after 3 turns. [2]

(iv) Show that a game can continue indefinitely. [1]

2 Answer this question on the insert provided.

(i) Use Dijkstra's algorithm to find the least weight route from A to G in the network shown in Fig. 2.1. Show the order in which you label vertices, give the route and its weight. [5]

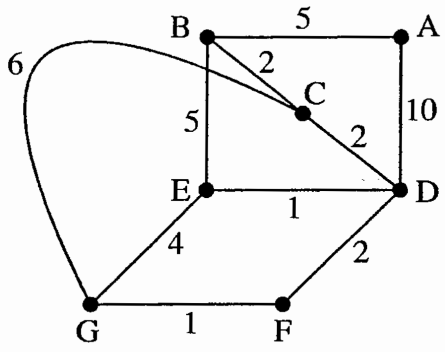

Fig. 2.1

(ii) Fig. 2.2 shows a partially completed application of Kruskal's algorithm to find a minimum spanning tree for the network.

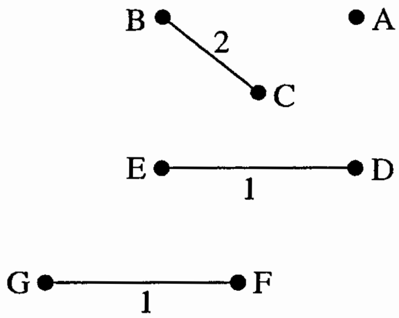

Fig. 2.2

Complete the algorithm and give the total weight of your minimum spanning tree. [3]

3 The following algorithm finds the highest common factor of two positive integers. ("int (x)" stands for the integer part of x, e.g. int (7.8) = 7.)

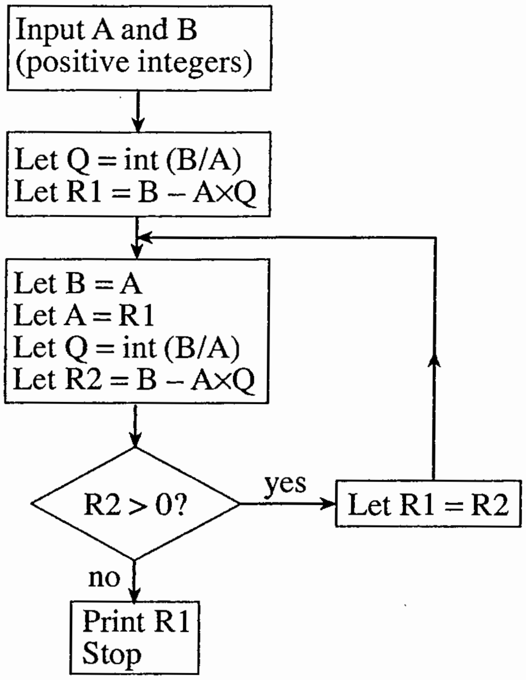

Fig. 3.1

(i) Run the algorithm with A = 84 and B = 660, showing all of your calculations. [3]

(ii) Run the algorithm with A = 660 and B = 84, showing as many calculations as are necessary. [2]

(iii) The algorithm is run with A = 30 and B = 42, and the result is shown in Table 3.2 below.

A	B	Q	R1	R2
30	42	1	12	
12	30	2		6
			6	
6	12	2		0

Print 6

Table 3.2

The first line of the table shows that $12 = 42 - 1 \times 30$.

Use the second line to obtain a similar expression for 6 in terms of 30 and 12.

Hence express 6 in the form $m \times 30 - n \times 42$, where m and n are integers. [3]

Section B

4 Answer this question on the insert provided.

The table shows activities involved in a "perm" in a hair salon, their durations and immediate predecessors.

	Activity	Duration (mins)	Immediate predecessor(s)
A	shampoo	5	–
B	prepare perm lotion	2	–
C	make coffee for customer	3	–
D	trim	5	A
E	clean sink	3	A
F	put rollers in	15	D
G	clean implements	3	D
H	apply perm lotion	5	B, F
I	leave to set	20	C, H
J	clean lotion pot and spreaders	3	H
K	neutralise and rinse	10	I, E
L	dry	10	K
M	wash up and clean up	15	K
N	style	4	G, L

Table 4

(i) Complete the activity-on-arc network in the insert to represent the precedences. [5]

(ii) Perform a forward pass and a backward pass to find early and late event times. Give the critical activities and the time needed to complete the perm. [6]

(iii) Give the total float time for the activity G. [1]

Activities D, F, H, K and N require a stylist.

Activities A, B, C, E, G, J and M are done by a trainee.

Activities I and L require no-one in attendance.

A stylist and a trainee are to give a perm to a customer.

(iv) Use the chart in the insert to show a schedule for the activities, assuming that all activities are started as early as possible. [3]

(v) Which activity would be better started at its latest start time? [1]

5 **There is an insert for use in parts (iii) and (iv) of this question.**

This question concerns the simulation of cars passing through two sets of pedestrian controlled traffic lights. The time intervals between cars arriving at the first set of lights are distributed according to Table 5.1.

Time interval (seconds)	2	5	15	25
Probability	$\frac{3}{7}$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Table 5.1

(i) Give an efficient rule for using two-digit random numbers to simulate arrival intervals. [3]

(ii) Use two-digit random numbers from the list below to simulate the arrival times of five cars at the first lights. The first car arrives at the time given by the first arrival interval.

Random numbers: 24, 01, 99, 89, 77, 19, 58, 42 [4]

The two sets of traffic lights are 23 seconds driving time apart. Moving cars are always at least 2 seconds apart. If there is a queue at a set of lights, then when the red light ends the first car in the queue moves off immediately, the second car 2 seconds later, the third 2 seconds after that, etc.

In this simple model there is to be no consideration of accelerations or decelerations, and the lights are either red or green.

Table 5.2 shows the times when the lights are red.

first set of lights	red start time	14	50	105	155
	red end time	29	65	120	170
second set of lights	red start time	10	55	105	150
	red end time	25	70	120	165

Table 5.2

(iii) Complete the table in the insert to simulate the passage of 10 cars through both sets of traffic lights. Use the arrival times given there. [5]

(iv) Find the mean delay experienced by these cars in passing through each set of lights. [3]

(v) How could the output from this simulation model be made more reliable? [1]

6 A recipe for jam states that the weight of sugar used must be between the weight of fruit used and four thirds of the weight of fruit used. Georgia has 10 kg of fruit available and 11 kg of sugar.

(i) Define two variables and formulate inequalities in those variables to model this information. [5]

(ii) Draw a graph to represent your inequalities. [5]

(iii) Find the vertices of your feasible region and identify the points which would represent the best mix of ingredients under each of the following circumstances.

(A) There is to be as much jam as possible, given that the weight of jam produced is the sum of the weights of the fruit and the sugar.

(B) There is to be as much jam as possible, given that it is to have the lowest possible proportion of sugar.

(C) There is to be as much jam as possible, given that it is to have the highest possible proportion of sugar.

(D) Fruit costs £1 per kg, sugar costs 50p per kg and the objective is to produce as much jam as possible within a budget of £15. [6]

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RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **4771**

Decision Mathematics 1

Monday **20 JUNE 2005** Morning 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Questions **1, 2** and **4**.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

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- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
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This question paper consists of 8 printed pages and an insert.

Section A (24 marks)

1 Answer this question on the insert provided.

The nodes in the unfinished graph in Fig. 1 represent six components, A, B, C, D, E, F and the mains. The components are to be joined by electrical cables to the mains. Each component can be joined directly to the mains, or can be joined via other components.

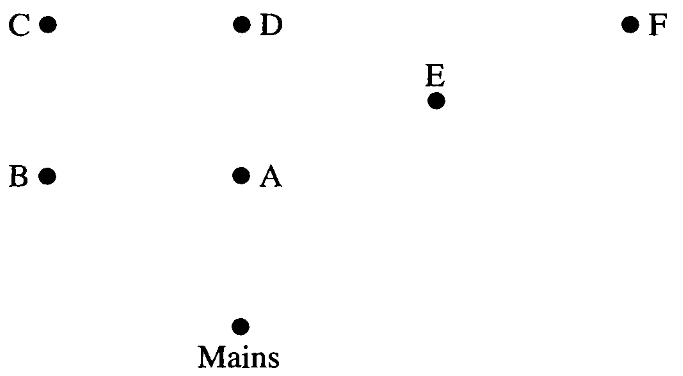

Fig. 1

The total number of connections that the electrician has to make is the sum of the orders of the nodes in the finished graph.

(i) Draw arcs representing suitable cables so that the electrician has to make as few connections as possible. Give this number. [3]

The electrician has a junction box. This can be represented by an eighth node on the graph.

(ii) What is the minimum number of connections which the electrician will have to make if he uses the junction box? [1]

(iii) The electrician has to make more connections if he uses his junction box. Why might he choose to use it anyway? [1]

The electrician decides not to use the junction box. He measures each of the distances between pairs of nodes, and records them in a complete graph. He then constructs a minimum connector for his graph to find which nodes to connect.

(iv) Will this result in the minimum number of connections? Justify your answer. [3]

2 Answer this question on the insert provided.

A maze is constructed by building east/west and north/south walls so that there is a route from the entrance to the exit. The maze is shown in Fig. 2.1.

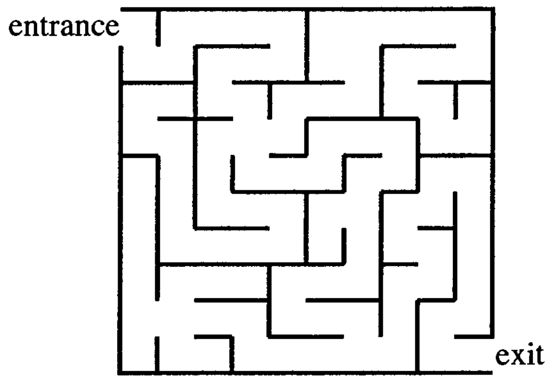

Fig. 2.1

On entering the maze Janet says "I'm always going to keep a hand in contact with the wall on the right." John says "I'm always going to keep a hand in contact with the wall on the left."

(i) On the insert for this question show Janet's route through the maze.

On the insert show John's route. [3]

(ii) Will these strategies always find a way through such mazes? Justify your answer. [2]

In some mazes the objective is to get to a marked point before exiting. An example is shown in Fig. 2.2, where **X** is the marked point.

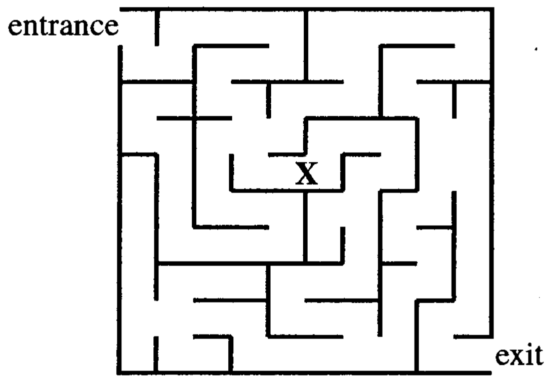

Fig. 2.2

In the maze shown in Fig. 2.2 Janet's algorithm takes her to **X**. John's algorithm does not take him to **X**. John modifies his algorithm by saying that he will turn his back on the exit if he arrives there without visiting **X**. He will then move onwards, continuing to keep a hand in contact with the wall on the left.

(iii) Will this modified algorithm take John to the point **X** in the maze in Fig. 2.2? [1]

(iv) Will this modified algorithm take John to **any** marked point in the maze in Fig. 2.2? Justify your answer. [2]

[Turn over

3 Table 3 gives the durations and immediate predecessors for the five activities of a project.

Activity	Duration (hours)	Immediate predecessor(s)
A	3	–
B	2	–
C	5	–
D	2	A
E	1	A, B

Table 3

(i) Draw an activity-on-arc network to represent the precedences. [2]

(ii) Find the early and late event times for the vertices of your network, and list the critical activities. [3]

(iii) Give the total and independent float for each activity which is not critical. [3]

Section B (48 marks)

4 Answer parts (i) and (ii) on the insert provided.

Fig. 4 shows a network of roads giving direct connections between a city, C, and 7 towns labelled P to V. The weights on the arcs are distances in kilometres. The local coastline is also shown.

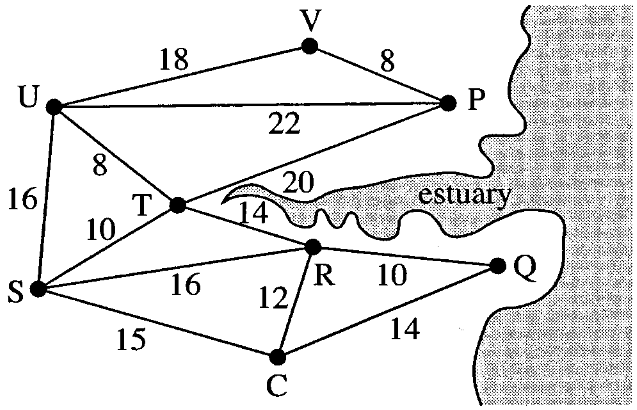

Fig. 4

(i) Use Dijkstra's algorithm on the insert to find the shortest distances from each of the towns to the city, C. List those distances and give the shortest route from P to C and from V to C. [8]

(ii) Use Kruskal's algorithm to find a minimum connector for the network. List the order in which you include arcs and give the length of your connector. [4]

A bridge is built giving a direct road between P and Q of length 12 km.

(iii) What effect does the bridge have on the shortest distances from the towns to the city? (You do not need to use an algorithm to answer this part of the question.) [1]

(iv) What effect does the bridge have on the minimum connector for the network? (You do not need to use an algorithm to answer this part of the question.) [1]

(v) Before the bridge was built it was possible to travel from P to C using every road once and only once. With the bridge in place, it is possible to travel from a different town to C using every road once and only once.

Give this town and justify your answer. [2]

[Turn over

5 A computer store has a stock of 10 laptops to lend to customers while their machines are being repaired. On any particular day the number of laptop loans requested follows the distribution given in Table 5.1.

Number requested	0	1	2	3	4
Probability	0.20	0.30	0.20	0.15	0.15

Table 5.1

(i) Give an efficient rule for using two-digit random numbers to simulate the daily number of requests for laptop loans. [2]

(ii) Use two-digit random numbers from the list below to simulate the number of loans requested on each of ten successive days.

Random numbers: 23, 02, 57, 80, 31, 72, 92, 78, 04, 07 [2]

The number of laptops returned from loan each day is modelled by the distribution given in Table 5.2, independently of the number on loan (which is always at least 5).

Number returned	0	1	2	3
Probability	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$

Table 5.2

(iii) Give an efficient rule for using two-digit random numbers to simulate the daily number of laptop returns. [2]

(iv) Use two-digit random numbers from the list below to simulate the number of returns on each of ten successive days.

Random numbers: 32, 98, 01, 32, 14, 21, 32, 71, 82, 54, 47 [2]

At the end of day 0 there are 7 laptops out on loan and 3 in stock. Each day returns are made in the morning and loans go out in the afternoon. If there is no laptop available the customer is disappointed and never gets a loaned laptop.

(v) Use your simulated numbers of requests and returns to simulate what happens over the next 10 days. For each day record the day number, the number of laptops in stock at the end of the day, and the number of customers that have to be disappointed. [3]

To try to avoid disappointing customers, if the number of laptops in stock at the end of a day is 2 or fewer, the store sends out e-mails to customers with loaned laptops asking for early return if possible. This changes the return distribution for the next day to that given in Table 5.3.

Number returned	0	1	2	3	4
Probability	0.1	0.1	0.4	0.2	0.2

Table 5.3

(vi) Simulate the 10 days again, but using this new policy.

Use the requests you produced in part **(ii)**. Use the random numbers given in part **(iv)** to simulate returns, but use either the distribution given in Table 5.2 or that given in Table 5.3, depending on the number of laptops in stock at the end of the previous day.

Is the new policy better? [5]

[Question 6 is printed overleaf.]

6 A company manufactures two types of potting compost, Flowerbase and Growmuch. The weekly amounts produced of each are constrained by the supplies of fibre and of nutrient mix. Each litre of Flowerbase requires 0.75 litres of fibre and 1 kg of nutrient mix. Each litre of Growmuch requires 0.5 litres of fibre and 2 kg of nutrient mix. There are 12 000 litres of fibre supplied each week, and 25 000 kg of nutrient mix.

The profit on Flowerbase is 9p per litre. The profit on Growmuch is 20p per litre.

(i) Formulate an LP to maximise the weekly profit subject to the constraints on fibre and nutrient mix. [6]

(ii) Solve your LP using a graphical approach. [6]

(iii) Consider each of the following separate circumstances.

(*A*) There is a reduction in the weekly supply of fibre from 12000 litres to 10000 litres. What effect does this have on profit? [1]

(*B*) The price of fibre is increased. Will this affect the optimal production plan? Justify your answer. [2]

(*C*) The supply of nutrient mix is increased to 30000 kg per week. What is the new profit? [1]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
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MEI STRUCTURED MATHEMATICS 4771

Decision Mathematics 1

Monday **23 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Questions **2**, **5** and **6**.
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HN/3

Section A (24 marks)

1 Table 1 shows a precedence table for a project.

Activity	Immediate predecessors	Duration (days)
A	-	5
B	-	3
C	A	3
D	A, B	4
E	A, B	5

Table 1

(i) Draw an activity-on-arc network to represent the precedences. [3]

(ii) Find the early event time and late event time for each vertex of your network, and list the critical activities. [3]

(iii) Extra resources become available which enable the durations of three activities to be reduced, each by up to two days. Which three activities should have their durations reduced so as to minimise the completion time of the project? What will be the new minimum project completion time? [2]

2 Answer this question on the insert provided.

An algorithm is specified in Fig. 2. It operates on two lists of numbers, each sorted into ascending order, to create a third list.

Step 1	Let A equal the first number in List 1. Delete the first number in List 1. Let B equal the first number in List 2. Delete the first number in List 2.
Step 2	If A $\leqslant$ B go to Step 3. Otherwise go to Step 4.
Step 3	Write A down at the end of List 3. If List 1 is not empty let A equal the first number in List 1, delete the first number in List 1 and go to Step 2. If List 1 is empty write down B at the end of List 3 and then copy the numbers in List 2 at the end of List 3. Then stop.
Step 4	Write B down at the end of List 3. If List 2 is not empty let B equal the first number in List 2, delete the first number in List 2 and go to Step 2. If List 2 is empty write down A at the end of List 3 and then copy the numbers in List 1 at the end of List 3. Then stop.

Fig. 2

(i) Complete the table in the insert showing the outcome of applying the algorithm to the following two lists:

List 1: 2, 34, 35, 56

List 2: 13, 22, 34, 81, 90, 92 [4]

(ii) What does the algorithm achieve? [1]

(iii) How many comparisons did you make in applying the algorithm? [1]

(iv) If the number of elements in List 1 is x, and the number of elements in List 2 is y, what is the maximum number of comparisons that will have to be made in applying the algorithm, and what is the minimum number? [2]

[Turn over

3 Fig. 3 shows a graph representing the seven bus journeys run each day between four rural towns. Each directed arc represents a single bus journey.

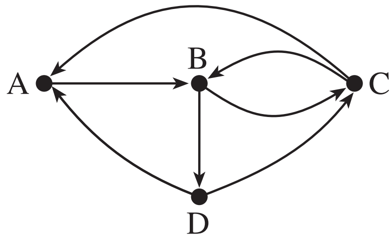

Fig. 3

(i) Show that if there is only one bus, which is in service at all times, then it must start at one town and end at a different town.

Give the start town and the end town. [3]

(ii) Show that there is only one Hamilton cycle in the graph.

Show that, if an extra journey is added from your end town to your start town, then there is still only one Hamilton cycle. [4]

(iii) A tourist is staying in town B. Give a route for her to visit every town by bus, visiting each town only once and returning to B. [1]

Section B (48 marks)

4 Table 4 shows the butter and sugar content in two recipes. The first recipe is for 1 kg of toffee and the second is for 1 kg of fudge.

	Toffee	Fudge
Butter	100 g	150 g
Sugar	800 g	700 g

Table 4

A confectioner has 1.5 kg of butter and 10 kg of sugar available. There are no constraints on the availability of other ingredients.

(i) What is the maximum amount of toffee which the confectioner could make? How much butter or sugar would be left over?

What is the maximum amount of fudge which the confectioner could make? How much butter or sugar would be left over? [4]

(ii) Formulate an LP to find the maximum total amount of toffee and fudge which the confectioner can make.

Solve your LP graphically. [8]

The confectioner charges £5.50 for 1 kg of toffee and £4.50 for 1 kg of fudge.

(iii) What quantities should he make to maximise his income? Justify your answer.

By how much would the price of toffee have to change for the maximum income solution to change? [4]

5 Answer this question on the insert provided.

Table 5 specifies a road network connecting 7 towns, A, B, … , G. The entries in Table 5 give the distances in miles between towns which are connected directly by roads.

	A	B	C	D	E	F	G
A	–	10	–	–	–	12	15
B	10	–	15	20	–	–	8
C	–	15	–	7	–	–	11
D	–	20	7	–	20	–	13
E	–	–	–	20	–	17	9
F	12	–	–	–	17	–	13
G	15	8	11	13	9	13	–

Table 5

(i) Using the copy of Table 5 in the insert, apply the tabular form of Prim's algorithm to the network, starting at vertex A. Show the order in which you connect the vertices.

Draw the resulting tree, give its total length and describe a practical application. [7]

(ii) The network in the insert shows the information in Table 5. Apply Dijkstra's algorithm to find the shortest route from A to E.

Give your route and its length. [6]

(iii) A tunnel is built through a hill between A and B, shortening the distance between A and B to 6 miles. How does this affect your answers to parts **(i)** and **(ii)**? [3]

6 Answer part (iv) of this question on the insert provided.

There are two types of customer who use the shop at a service station. 70% buy fuel, the other 30% do not. There is only one till in operation.

(i) Give an efficient rule for using one-digit random numbers to simulate the type of customer arriving at the service station. [1]

Table 6.1 shows the distribution of time taken at the till by customers who are buying fuel.

Time taken (mins)	1	1.5	2	2.5
Probability	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{10}$

Table 6.1

(ii) Specify an efficient rule for using one-digit random numbers to simulate the time taken at the till by customers purchasing fuel. [2]

Table 6.2 shows the distribution of time taken at the till by customers who are not buying fuel.

Time taken (mins)	1	1.5	2	2.5	3
Probability	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Table 6.2

(iii) Specify an efficient rule for using two-digit random numbers to simulate the time taken at the till by customers not buying fuel.

What is the advantage in using two-digit random numbers instead of one-digit random numbers in this part of the question? [4]

The table in the insert shows a partially completed simulation study of 10 customers arriving at the till.

(iv) Complete the table using the random numbers which are provided. [7]

(v) Calculate the mean total time spent queuing and paying. [2]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4771

Decision Mathematics 1

Thursday **15 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Questions **1** and **6**.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

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Section A (24 marks)

1 Answer this question on the insert provided.

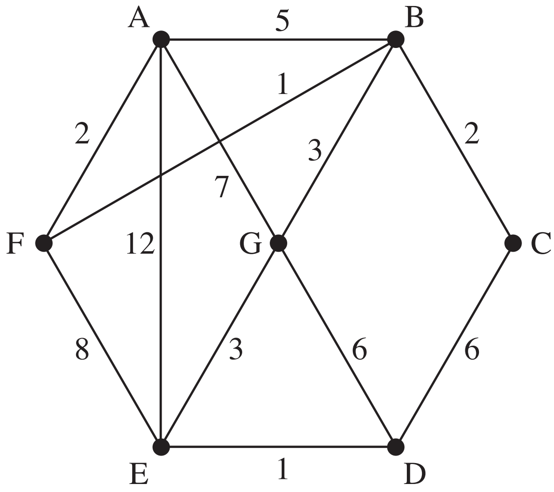


Fig. 1

(i) Apply Dijkstra's algorithm to the copy of Fig. 1 in the insert to find the least weight route from A to D.

Give your route and its weight. [6]

(ii) Arc DE is now deleted. Write down the weight of the new least weight route from A to D, and explain how your working in part **(i)** shows that it is the least weight. [2]

2 Fig. 2.1 represents the two floors of a house. There are 5 rooms shown, plus a hall and a landing, which are to be regarded as separate rooms. Each "×" represents an internal doorway connecting two rooms. The "f" represents the staircase, connecting the hall and the landing.

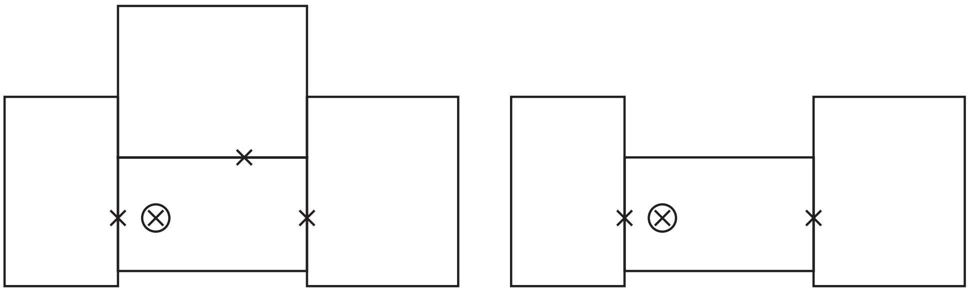


Fig. 2.1

(i) Draw a graph representing this information, with vertices representing rooms, and arcs representing internal connections (doorways and the stairs).

What is the name of the type of graph of which this is an example? [3]

(ii) A larger house has 12 rooms on two floors, plus a hall and a landing. Each ground floor room has a single door, which leads to the hall. Each first floor room has a single door, which leads to the landing. There is a single staircase connecting the hall and the landing.

How many arcs are there in the graph of this house? [1]

(iii) Another house has 12 rooms on three floors, plus a hall, a first floor landing and a second floor landing. Again, each room has a single door on to the hall or a landing. There is one staircase from the hall to the first floor landing, and another staircase joining the two landings.

How many arcs are there in the graph of this house? [1]

(iv) Fig. 2.2 shows the graph of another two-floor house. It has 8 rooms plus a hall and a landing. There is a single staircase.

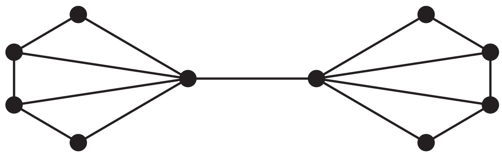


Fig. 2.2

Draw a possible floor plan, showing internal connections. [3]

3 An incomplete algorithm is specified in Fig. 3.

$f(x) = x^2 - 2$

Initial values: $L = 0, R = 2$.

Step 1 Compute $M = \dfrac{L+R}{2}$.

Step 2 Compute $f(M)$.

Step 3 If $f(M) < 0$ change the value of L to that of M.

Otherwise change the value of R to that of M.

Step 4 Go to Step 1.

Fig. 3

(i) Apply two iterations of the algorithm. [6]

(ii) After 10 iterations $L = 1.414063, R = 1.416016, M = 1.416016$ and $f(M) = 0.005100$.

Say what the algorithm achieves. [1]

(iii) Say what is needed to complete the algorithm. [1]

Section B (48 marks)

4 Table 4.1 shows some of the activities involved in preparing for a meeting.

	Activity	Duration (hours)	Immediate predecessors
A	Agree date	1	–
B	Construct agenda	0.5	–
C	Book venue	0.25	A
D	Order refreshments	0.25	C
E	Inform participants	0.5	B, C

Table 4.1

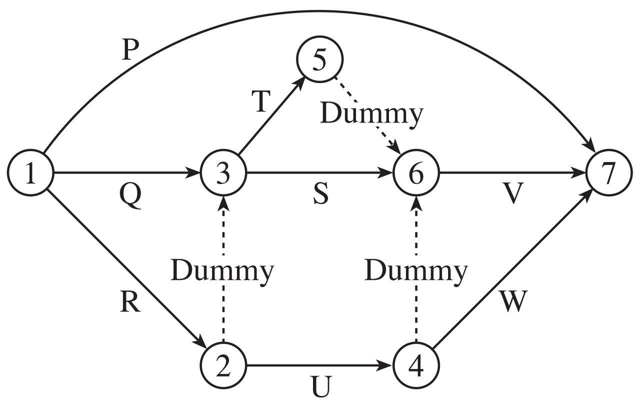
(i) Draw an activity-on-arc network to represent the precedences. [4]

(ii) Find the early event time and the late event time for each vertex of your network, and list the critical activities. [3]

(iii) Assuming that each activity requires one person and that each activity starts at its earliest start time, draw a resource histogram. [2]

(iv) In fact although activity A has duration 1 hour, it actually involves only 0.5 hours work, since 0.5 hours involves waiting for replies. Given this information, and the fact that there is only one person available to do the work, what is the shortest time needed to prepare for the meeting? [2]

Fig. 4.2 shows an activity network for the tasks which have to be completed after the meeting.



P: Clean room
Q: Prepare draft minutes
R: Allocate action tasks
S: Circulate draft minutes
T: Approve task allocations
U: Obtain budgets for tasks
V: Post minutes
W: Pay refreshments bill

Fig. 4.2

(v) Draw a precedence table for these activities. [5]

5 John is reviewing his lifestyle, and in particular his leisure commitments. He enjoys badminton and squash, but is not sure whether he should persist with one or both. Both cost money and both take time.

Playing badminton costs £3 per hour and playing squash costs £4 per hour. John has £11 per week to spend on these activities.

John takes 0.5 hours to recover from every hour of badminton and 0.75 hours to recover from every hour of squash. He has 5 hours in total available per week to play and recover.

(i) Define appropriate variables and formulate two inequalities to model John's constraints. [3]

(ii) Draw a graph to represent your inequalities.

Give the coordinates of the vertices of your feasible region. [6]

(iii) John is not sure how to define an objective function for his problem, but he says that he likes squash "twice as much" as badminton. Letting every hour of badminton be worth one "satisfaction point" define an objective function for John's problem, taking into account his "twice as much" statement. [1]

(iv) Solve the resulting LP problem. [2]

(v) Given that badminton and squash courts are charged by the hour, explain why the solution to the LP is not a feasible solution to John's practical problem. Give the best feasible solution. [2]

(vi) If instead John had said that he liked badminton more than squash, what would have been his best feasible solution? [2]

6 **Answer parts (ii)(*A*) and (iii)(*B*) of this question on the insert provided.**

A particular component of a machine sometimes fails. The probability of failure depends on the age of the component, as shown in Table 6.

Year of life	first	second	third	fourth	fifth	sixth
Probability of failure during year, given no earlier failure	0.10	0.05	0.02	0.20	0.20	0.30

Table 6

You are to simulate six years of machine operation to estimate the probability of the component failing during that time. This will involve you using six 2-digit random numbers, one for each year.

(i) Give a rule for using a 2-digit random number to simulate failure of the component in its first year of life.

Similarly give rules for simulating failure during each of years 2 to 6. [3]

(ii) (*A*) Use your rules, together with the random numbers given in the insert, to complete the simulation table in the insert. This simulates 10 repetitions of six years operation of the machine. Start in the first column working down cell-by-cell. In each cell enter a tick if there is no simulated failure and a cross if there is a simulated failure.

Stop and move on to the next column if a failure occurs. [5]

(*B*) Use your results to estimate the probability of a failure occurring. [1]

It is suggested that any component that has not failed during the first three years of its life should automatically be replaced.

(iii) (*A*) Describe how to simulate the operation of this policy. [2]

(*B*) Use the table in the insert to simulate 10 repetitions of the application of this policy. Re-use the same random numbers that are given in the insert. [3]

(*C*) Use your results to estimate the probability of a failure occurring. [1]

(iv) How might the reliability of your estimates in parts **(ii)** and **(iii)** be improved? [1]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4771/01**

MATHEMATICS (MEI)

Decision Mathematics 1

TUESDAY 23 JANUARY 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Printed Answer Book
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the printed answer book.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

  [Turn over

Answer all the questions in the printed answer book provided.

Section A (24 marks)

1 Each of the following symbols consists of boundaries enclosing regions.

0 1 2 3 4 5 6 7 8 9

The symbol representing zero has three regions, the outside, that between the two boundaries and the inside.

To classify the symbols a graph is produced for each one. The graph has a vertex for each region, with arcs connecting regions which share a boundary. Thus the graph for

0 is .

(i) Produce the graph for the symbol 4. [1]

(ii) Give two symbols each having the graph . [2]

(iii) Produce the graph for the symbol 8. [2]

(iv) Produce a single graph for the composite symbol 88. [2]

(v) Give the name of a connected graph with n nodes and $n - 1$ arcs. [1]

2 The following algorithm is a version of bubble sort.

Step 1 Store the values to be sorted in locations L(1), L(2), … , L(n) and set i to be the number, n, of values to be sorted.

Step 2 Set j = 1.

Step 3 Compare the values in locations L(j) and L(j+1) and swap them if that in L(j) is larger than that in L(j+1).

Step 4 Add 1 to j.

Step 5 If j is less than i then go to step 3.

Step 5 Write out the current list, L(1), L(2), … , L(n).

Step 6 Subtract 1 from i.

Step 7 If i is larger than 1 then go to step 2.

Step 8 Stop.

(i) Apply this algorithm to sort the following list.

109 32 3 523 58.

Count the number of comparisons and the number of swaps which you make in applying the algorithm. [4]

(ii) Put the five values into the order which maximises the number of swaps made in applying the algorithm, and give that number. [2]

(iii) Bubble sort has quadratic complexity. Using bubble sort it takes a computer 1.5 seconds to sort a list of 1000 values. Approximately how long would it take to sort a list of 100 000 values? (Give your answer in hours and minutes.) [2]

3 A bag contains five pieces of paper labelled A, B, C, D and E. One piece is drawn at random from the bag. If the piece is labelled with a vowel (A or E) then the process stops. Otherwise the piece of paper is replaced, the bag is shaken, and the process is repeated. You are to simulate this process to estimate the mean number of draws needed to get a vowel.

(i) Show how to use single digit random numbers to simulate the process efficiently. You need to describe exactly how your simulation will work. [3]

(ii) Use the random numbers in your answer book to run your simulation 5 times, recording your results. [2]

(iii) From your results compute an estimate of the mean number of draws needed to get a vowel. [2]

(iv) State how you could produce a more accurate estimate. [1]

Section B (48 marks)

4 Cassi is managing the building of a house. The table shows the major activities that are involved, their durations and their precedences.

Activity		Duration (days)	Immediate predecessors
A	Build concrete frame	10	–
B	Lay bricks	7	A
C	Lay roof tiles	10	A
D	First fit electrics	5	B
E	First fit plumbing	4	B
F	Plastering	6	C, D, E
G	Second fit electrics	3	F
H	Second fit plumbing	2	F
I	Tiling	10	G, H
J	Fit sanitary ware	2	H
K	Fit windows and doors	5	I

(i) Draw an activity-on-arc network to represent this information. [5]

(ii) Find the early time and the late time for each event. Give the project duration and list the critical activities. [6]

(iii) Calculate total and independent floats for each non-critical activity. [2]

Cassi's clients wish to take delivery in 42 days. Some durations can be reduced, at extra cost, to achieve this.

- The tiler will finish activity I in 9 days for an extra £250, or in 8 days for an extra £500.
- The bricklayer will cut his total of 7 days on activity B by up to 3 days at an extra cost of £350 per day.
- The electrician could be paid £300 more to cut a day off activity D, or £600 more to cut two days.

(iv) What is the cheapest way in which Cassi can get the house built in 42 days? [3]

5 Leone is designing her new garden. She wants to have at least 1000 m^2, split between lawn and flower beds.

Initial costs are £0.80 per m^2 for lawn and £0.40 per m^2 for flowerbeds. Leone's budget is £500.

Leone prefers flower beds to lawn, and she wants the area for flower beds to be at least twice the area for lawn. However, she wants to have at least 200 m^2 of lawn.

Maintenance costs each year are £0.15 per m^2 for lawn and £0.25 per m^2 for flower beds. Leone wants to minimize the maintenance costs of her garden.

(i) Formulate Leone's problem as a linear programming problem. [7]

(ii) Produce a graph to illustrate the inequalities. [6]

(iii) Solve Leone's problem. [2]

(iv) If Leone had more than £500 available initially, how much extra could she spend to minimize maintenance costs? [1]

6 In a factory a network of pipes connects 6 vats, A, B, C, D, E and F. Two separate connectors need to be chosen from the network The table shows the lengths of pipes (metres) connecting the 6 vats.

	A	B	C	D	E	F
A	–	7	–	–	12	–
B	7	–	5	3	6	6
C	–	5	–	8	4	7
D	–	3	8	–	1	5
E	12	6	4	1	–	7
F	–	6	7	5	7	–

(i) Use Kruskal's algorithm to find a minimum connector. Show the order in which you select pipes, draw your connector and give its total length. [5]

(ii) Produce a new table excluding the pipes which you selected in part **(i)**. Use the tabular form of Prim's algorithm to find a second minimum connector from this reduced set of pipes. Show your working, draw your connector and give its total length. [7]

(iii) The factory manager prefers the following pair of connectors:

$$\{AB, BC, BD, BE, BF\} \text{ and } \{AE, BF, CE, DE, DF\}.$$

Give two possible reasons for this preference. [4]

RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4771/01**

MATHEMATICS (MEI)

Decision Mathematics 1

MONDAY 18 JUNE 2007

Morning
Time: 1 hour 30 minutes

Additional materials:
Printed Answer Book
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the printed answer book.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

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Answer all the questions in the printed answer book provided.

Section A (24 marks)

1 Bus routes connect towns A and B and towns A and C.

Train lines connect towns B and D, towns C and D, and towns A and C.

John represents this information in a graph with four nodes, one for each town, in which an arc is drawn for each connection, giving five arcs in all.

(i) Draw John's graph. [2]

(ii) Is John's graph simple? Justify your answer. [2]

Jamil represents the information in a graph with five nodes. He uses one node for each of towns A, B and D. Because in town C the bus station and train station are some distance apart, he uses a node labelled C(bus) and a node labelled C(train). Again there are 5 arcs, each representing a connection.

(iii) Draw Jamil's graph. [2]

(iv) Is Jamil's graph a tree? Justify your answer. [2]

2 Two hikers each have a 25 litre rucksack to pack. The items to be packed have volumes of 14, 6, 11, 9 and 6 litres.

(i) Apply the first fit algorithm to the items in the order given and comment on the outcome. [3]

(ii) Write the five items in descending order of volume. Apply the first fit decreasing algorithm to find a packing for the rucksacks. [3]

(iii) The hikers argue that the first fit decreasing algorithm does not produce a fair allocation of volumes to rucksacks. Produce a packing which gives a fairer allocation of volumes between the two rucksacks. Explain why the hikers might **not** want to use this packing. [2]

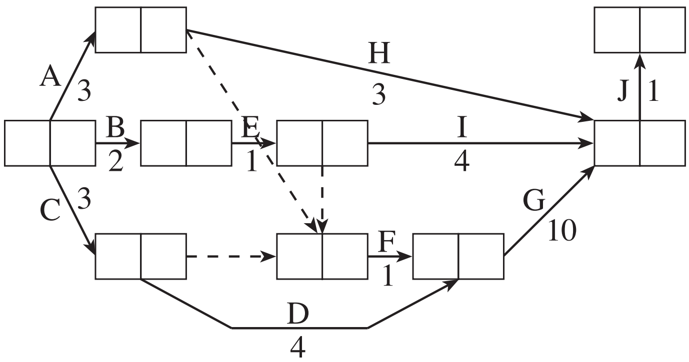
3 Use a graphical approach to solve the following LP. [8]

$$\begin{array}{ll} \text{Maximise} & 2x+3y \\ \text{subject to} & x+5y \leqslant 14 \\ & x+2y \leqslant 8 \\ & 3x+y \leqslant 21 \\ & x \geqslant 0 \\ & y \geqslant 0 \end{array}$$

Section B (48 marks)

4 Colin is setting off for a day's sailing. The table and the activity network show the major activities that are involved, their durations and their precedences.

A	Rig foresail
B	Lower sprayhood
C	Start engine
D	Pump out bilges
E	Rig mainsail
F	Cast off mooring ropes
G	Motor out of harbour
H	Raise foresail
I	Raise mainsail
J	Stop engine and start sailing



(i) Complete the table in your answer book showing the immediate predecessors for each activity. [3]

(ii) Find the early time and the late time for each event. Give the project duration and list the critical activities. [6]

When he sails on his own Colin can only do one thing at a time with the exception of activity G, motoring out of the harbour.

(iii) Use the activity network to determine which activities Colin can perform whilst motoring out of the harbour. [1]

(iv) Find the minimum time to complete the activities when Colin sails on his own, and give a schedule for him to achieve this. [2]

(v) Find the minimum time to complete the activities when Colin sails with one other crew member, and give a schedule for them to achieve this. [4]

5 The table shows the weights on the arcs of a network.

	A	B	C	D	E	F	G
A	–	11	–	–	10	3	5
B	11	–	8	5	–	–	14
C	–	8	–	2	–	7	–
D	–	5	2	–	6	–	–
E	10	–	–	6	–	6	–
F	3	–	7	–	6	–	–
G	5	14	–	–	–	–	–

(i) Draw the network. [3]

(ii) Apply Dijkstra's algorithm to find the least weight route from G to D. (Do this on the network you drew for part **(i)**.)

Give your route and its total weight. [6]

(iii) Find by inspection the route from G to D such that the minimum of the weights for arcs on the route is as large as possible. Give your route and its minimum arc weight. Give an application in which this might be needed. [3]

(iv) Consider how Dijkstra's algorithm could be modified to solve the problem in part **(iii)**. Explain how to update working values. Explain how to select the next vertex to be permanently labelled. [4]

6 In winter in Metland the weather each day can be classified as dry, wet or snowy. The table shows the probabilities for the next day's weather given the current day's weather.

		next day's weather		
		dry	wet	snowy
current day's weather	dry	$\frac{4}{10}$	$\frac{3}{10}$	$\frac{3}{10}$
	wet	$\frac{2}{10}$	$\frac{5}{10}$	$\frac{3}{10}$
	snowy	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{3}{7}$

You are to use two-digit random numbers to simulate the winter weather in Metland.

(i) Give an efficient rule for using two-digit random numbers to simulate tomorrow's weather if today is

(*A*) dry, [2]

(*B*) wet, [2]

(*C*) snowy. [3]

(ii) Today is a dry winter's day in Metland. Use the following two-digit random numbers to simulate the next 7 days' weather in Metland. [4]

23 85 98 99 56 47 82 14 03 12

(iii) Use your simulation from part **(ii)** to estimate the proportion of dry days in a Metland winter. [1]

(iv) Explain how you could use simulation to produce an improved estimate of the proportion of dry days in a Metland winter. [2]

(v) Give two criticisms of this model of weather. [2]

ADVANCED SUBSIDIARY GCE UNIT **4771/01**

MATHEMATICS (MEI)

Decision Mathematics 1

TUESDAY 22 JANUARY 2008

Afternoon

Time: 1 hour 30 minutes

CUP/T38511

Additional materials: Printed Answer Book (Enclosed)
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks for each question is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **7** printed pages and **1** blank page.

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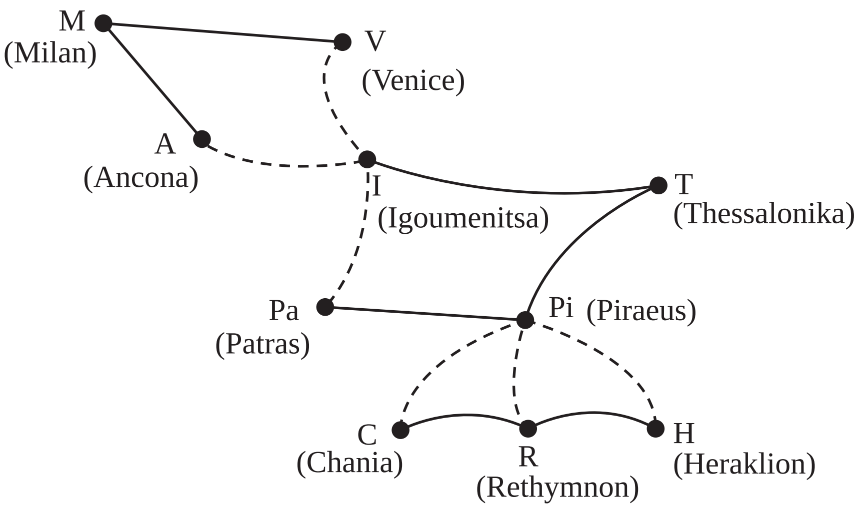
Answer all the questions in the printed answer book provided.

Section A (24 marks)

1 The graph shows routes that are available to an international lorry driver. The solid arcs represent motorways and the broken arcs represent ferry crossings.



(i) Give a route from Milan to Chania involving exactly two ferry crossings.

How many such routes are there? [2]

(ii) Give a route from Milan to Chania involving exactly three ferry crossings.

How many such routes are there? [2]

(iii) Give a route from Milan to Chania using as many ferry crossings as possible, without repeating any arc. [1]

(iv) Give a route leaving Piraeus and finishing elsewhere which uses every arc once and only once. [3]

2 Consider the following linear programming problem.

Maximise $P = 6x + 7y$

subject to
$2x + 3y \leqslant 9$
$3x + 2y \leqslant 12$
$x \geqslant 0$
$y \geqslant 0$

(i) Use a graphical approach to solve the problem. [6]

(ii) Give the optimal values of x, y and P when x and y are integers. [2]

3 The following algorithm (J. M. Oudin, 1940) claims to compute the date of Easter Sunday in the Gregorian calendar system.
The algorithm uses the year, y, to give the month, m, and day, d, of Easter Sunday.
All variables are integers and **all remainders from division are dropped**. For example, 7 divided by 3 is 2 remainder 1. The remainder is dropped, giving the answer 2.

$c = y / 100$

$n = y - 19 \times (y / 19)$

$k = (c - 17) / 25$

$i = c - (c / 4) - (c - k) / 3 + (19 \times n) + 15$

$i = i - 30 \times (i / 30)$

$i = i - (i / 28) \times (1 - (i / 28) \times (29 / (i + 1)) \times ((21 - n) / 11))$

$j = y + (y / 4) + i + 2 - c + (c / 4)$

$j = j - 7 \times (j / 7)$

$p = i - j$

$m = 3 + (p + 40) / 44$

$d = p + 28 - 31 \times (m / 4)$

For example, for 2008:

$y = 2008$

$c = 2008/100 = 20$

$n = 2008 - 19 \times (2008/19) = 2008 - 19 \times (105) = 13$, etc.

Complete the calculation for 2008. [8]

Section B (48 marks)

4 In a population colonizing an island 40% of the first generation (parents) have brown eyes, 40% have blue eyes and 20% have green eyes. Offspring eye colour is determined according to the following rules.

	Eye colours of parents	Eye colour of offspring
1	both brown	brown
2	one brown and one blue	50% brown and 50% blue
3	one brown and one green	blue
4	both blue	25% brown, 50% blue and 25% green
5	one blue and one green	50% blue and 50% green
6	both green	green

(i) Give an efficient rule for using 1-digit random numbers to simulate the eye colour of a parent randomly selected from the colonizing population. [3]

(ii) Give an efficient rule for using 1-digit random numbers to simulate the eye colour of offspring born of parents both of whom have blue eyes. [4]

The table in your answer book shows an incomplete simulation in which parent eye colours have been randomly selected, but in which offspring eye colours remain to be determined or simulated.

(iii) Complete the table using the given random numbers where needed. (You will need your own rules for cases 2 and 5.)
Each time you use a random number, explain how you decide which eye colour for the offspring. [9]

5 The table shows some of the activities involved in building a block of flats. The table gives their durations and their immediate predecessors.

Activity		Duration (weeks)	Immediate Predecessors
A	Survey sites	8	–
B	Purchase land	22	A
C	Supply materials	10	–
D	Supply machinery	4	–
E	Excavate foundations	9	B, D
F	Lay drains	11	B, C, D
G	Build walls	9	E, F
H	Lay floor	10	E, F
I	Install roof	3	G
J	Install electrics	5	G

(i) Draw an activity on arc network for these activities. [4]

(ii) Mark on your diagram the early and late times for each event. Give the minimum completion time and the critical activities. [6]

Each of the tasks E, F, H and J can be speeded up at extra cost. The maximum number of weeks by which each task can be shortened, and the extra cost for each week that is saved, are shown in the table below.

Task	E	F	H	J
Maximum number of weeks by which task may be shortened	3	3	1	3
Cost per week of shortening task (in thousands of pounds)	30	15	6	20

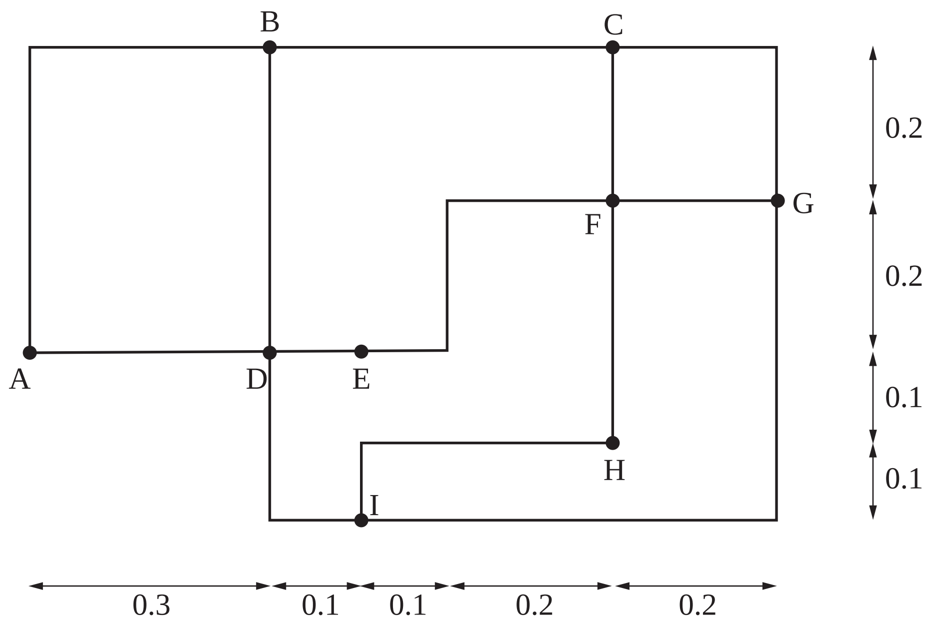
(iii) Find the new shortest time for the flats to be completed. [1]

(iv) List the activities which will need to be speeded up to achieve the shortest time found in part **(iii)**, and the times by which each must be shortened. [4]

(v) Find the total extra cost needed to achieve the new shortest time. [1]

[**Question 6 is printed overleaf.**]

6 The diagram shows routes between points in a town. The distances are in kilometres.



(i) Use an appropriate algorithm to find a set of connecting arcs of minimum total length. Indicate your connecting arcs on the copy of the diagram in your answer book, and give their total length. [7]

(ii) Give the name of the algorithm you have used, and describe it briefly. [2]

(iii) Using the second diagram in your answer book, apply Dijkstra's algorithm to find the shortest distances from A to each of the other points.

List the connections that are used, and give their total length. [7]

RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

4771/01

MATHEMATICS (MEI)

Decision Mathematics 1

THURSDAY 12 JUNE 2008

Morning
Time: 1 hour 30 minutes

Additional materials: Printed Answer Book (enclosed)
MEI Examination Formulae and Tables (MF2)

CUP/T44383

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
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INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **8** printed pages.

Answer all the questions in the printed answer book provided.

Section A (24 marks)

1 Consider the following LP.

Maximise $x + y$

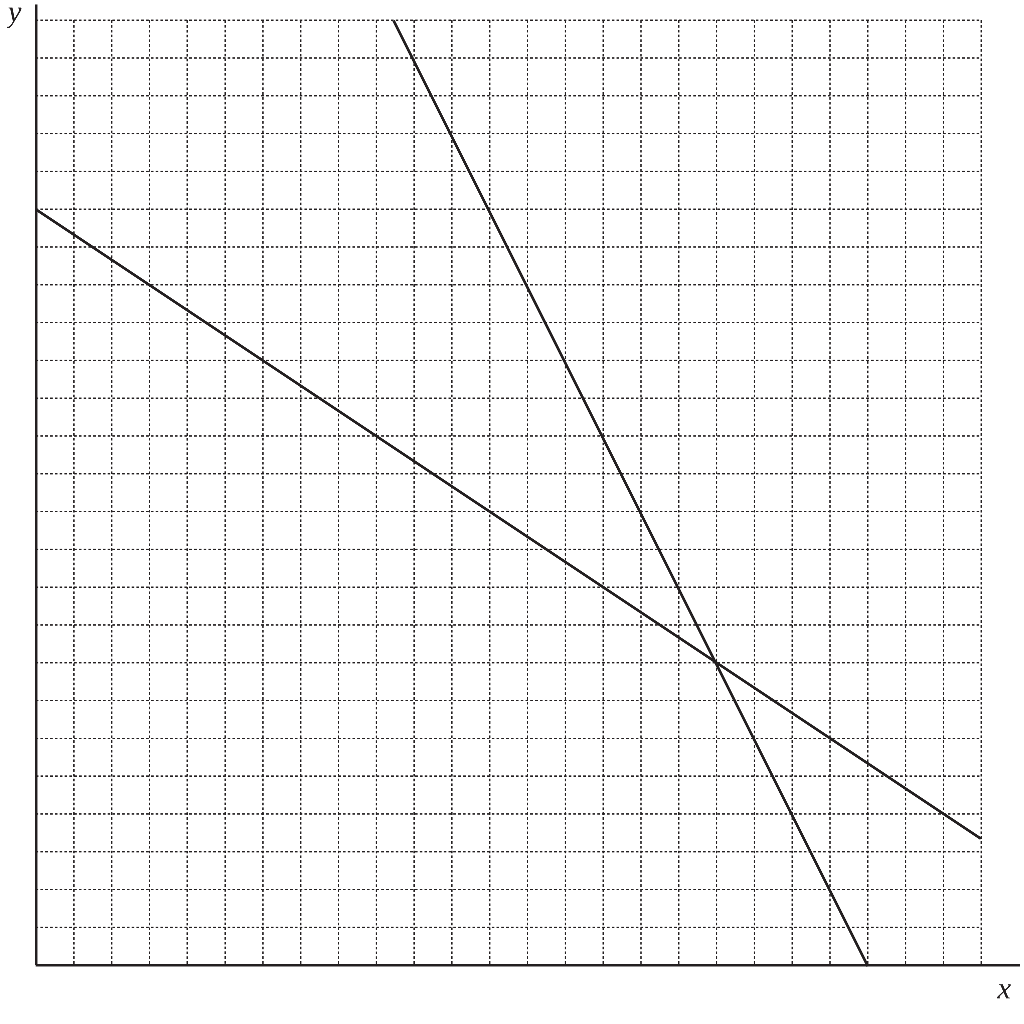
subject to $2x + y < 44$

$2x + 3y < 60$

$10x + 11y < 244$

Part of a graphical solution is produced below and in your answer book.

Complete this graphical solution in your answer book. [8]



2 The following algorithm acts on a list of three or more numbers.

Step 1: Set both X and Y equal to the first number on the list.

Step 2: If there is no next number then go to Step 5.

Step 3: If the next number on the list is bigger than X then set X equal to it. If it is less than Y then set Y equal to it.

Step 4: Go to Step 2.

Step 5: Delete a number equal to X from the list and delete a number equal to Y from the list.

Step 6: If there is one number left then record it as the answer and stop.

Step 7: If there are two numbers left then record their mean as the answer and stop.

Step 8: Go to Step 1.

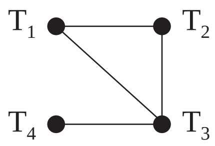
(i) Apply the algorithm to the list 5, 14, 153, 6, 24, 2, 14, 15, counting the number of comparisons which you have to make. [3]

(ii) Apply the algorithm to the list 5, 14, 153, 6, 24, 2, 14, counting the number of comparisons which you have to make. [3]

(iii) Say what the algorithm is finding. [1]

(iv) The order of the algorithm is quadratic. Explain what this means when it is applied to long lists. [1]

3 The graph represents four towns together with (two-way) roads connecting them.



A *path* is a set of connected arcs linking one vertex to another. A path contains no repeated vertex.

$T_1 \rightarrow T_2$ and $T_1 \rightarrow T_3 \rightarrow T_2$ are paths.

(i) There are six paths from T_1. List them. [2]

(ii) List the paths from T_4. [2]

(iii) How many paths are there altogether? [2]

For this question a *route* is defined to be a path in which the direction of the arcs is not relevant. Thus $T_1 \rightarrow T_2$ and $T_2 \rightarrow T_1$ are the same route. Similarly $T_1 \rightarrow T_3 \rightarrow T_2$ and $T_2 \rightarrow T_3 \rightarrow T_1$ are the same route (but note that $T_1 \rightarrow T_2 \rightarrow T_3$ is different).

(iv) How many routes are there altogether? [2]

Section B (48 marks)

4 Joe is to catch a plane to go on holiday. He has arranged to leave his car at a car park near to the airport. There is a bus service from the car park to the airport, and the bus leaves when there are at least 15 passengers on board. Joe is delayed getting to the car park and arrives needing the bus to leave within 15 minutes if he is to catch his plane. He is the 10th passenger to board the bus, so he has to wait for another 5 passengers to arrive.

The distribution of the time intervals between car arrivals and the distribution of the number of passengers per car are given below.

Time interval between cars (minutes)	1	2	3	4	5
Probability	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{1}{10}$

Number of passengers per car	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{12}$

(i) Give an efficient rule for using 2-digit random numbers to simulate the intervals between car arrivals. [3]

(ii) Give an efficient rule for using 2-digit random numbers to simulate the number of passengers in a car. [4]

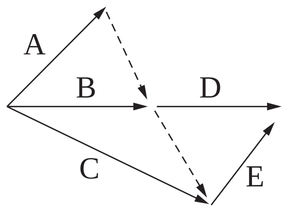
(iii) The incomplete table in your answer book shows the results of nine simulations of the situation. Complete the table, showing in each case whether or not Joe catches his plane. [3]

(iv) Use the random numbers provided in your answer book to run a tenth simulation. [4]

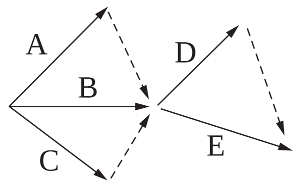
(v) Estimate the probability of Joe catching his plane. State how you could improve your estimate. [2]

5 **(a)** The graphs below illustrate the precedences involved in running two projects, each consisting of the same activities A, B, C, D and E.

Project 1



Project 2



(i) For one activity the precedences in the two projects are different. State which activity and describe the difference. [3]

(ii) The table below shows the durations of the five activities.

Activity	A	B	C	D	E
Duration	2	1	x	3	2

Give the total time for project 1 for all possible values of x.
Give the total time for project 2 for all possible values of x. [3]

(b) The durations and precedences for the activities in a project are shown in the table.

Activity	Duration	Immediate predecessors
R	2	–
S	1	–
T	5	–
W	3	R, S
X	2	R, S, T
Y	3	R
Z	1	W, Y

(i) Draw an activity on arc network to represent this information. [4]

(ii) Find the early time and the late time for each event. Give the project duration and list the critical activities. [6]

6 The matrix gives the lengths of the arcs of a network.

	A	B	C	D	E	F
A	–	10	7	–	9	5
B	10	–	–	1	–	4
C	7	–	–	–	3	–
D	–	1	–	–	2	–
E	9	–	3	2	–	–
F	5	4	–	–	–	–

(i) Using the copy of the matrix in your answer book, apply the tabular form of Prim's algorithm to find a minimum connector for the network. Start by choosing vertex A and show the order in which you include vertices.
List the arcs in your connector and give its total length. [6]

Serena takes a different approach to find a minimum connector. She first uses Dijkstra's algorithm to find shortest paths from A to each of the other vertices. She then uses the arcs in those paths to construct a connector.

(ii) Draw the network using the vertices printed in your answer book. [2]

(iii) Apply Serena's method to produce a connector.
List the arcs in the connector and give its total length. [6]

Serena adapts her method by starting from each vertex in turn, producing six connectors, from which she chooses the best.

(iv) Serena's approach will not find the minimum connector in all networks, but it is an algorithm. What is its algorithmic complexity? Justify your answer. [2]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) **4771**

Decision Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- Printed Answer Book (inserted)
- * MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 19 January 2009
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the printed Answer Book.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- This document consists of **8** pages. Any blank pages are indicated.

Answer all questions in the printed answer book provided.

Section A (24 marks)

1 Alfred, Ben, Charles and David meet, and some handshaking takes place.

- Alfred shakes hands with David.
- Ben shakes hands with Charles and David.
- Charles shakes hands with Ben and David.

(i) Complete the bipartite graph in your answer book showing A (Alfred), B (Ben), C (Charles) and D (David), and the number of people each shakes hands with. **[4]**

(ii) Explain why, whatever handshaking takes place, the resulting bipartite graph cannot contain both an arc terminating at 0 and another arc terminating at 3. **[2]**

(iii) Explain why, whatever number of people meet, and whatever handshaking takes place, there must always be two people who shake hands with the same number of people. **[2]**

2 The following algorithm computes the number of comparisons made when Prim's algorithm is applied to a complete network on n vertices ($n > 2$).

Step 1 Input the value of n
Step 2 Let $i = 1$
Step 3 Let $j = n - 2$
Step 4 Let $k = j$
Step 5 Let $i = i + 1$
Step 6 Let $j = j - 1$
Step 7 Let $k = k + (i \times j) + (i - 1)$
Step 8 If $j > 0$ then go to Step 5
Step 9 Print k
Step 10 Stop

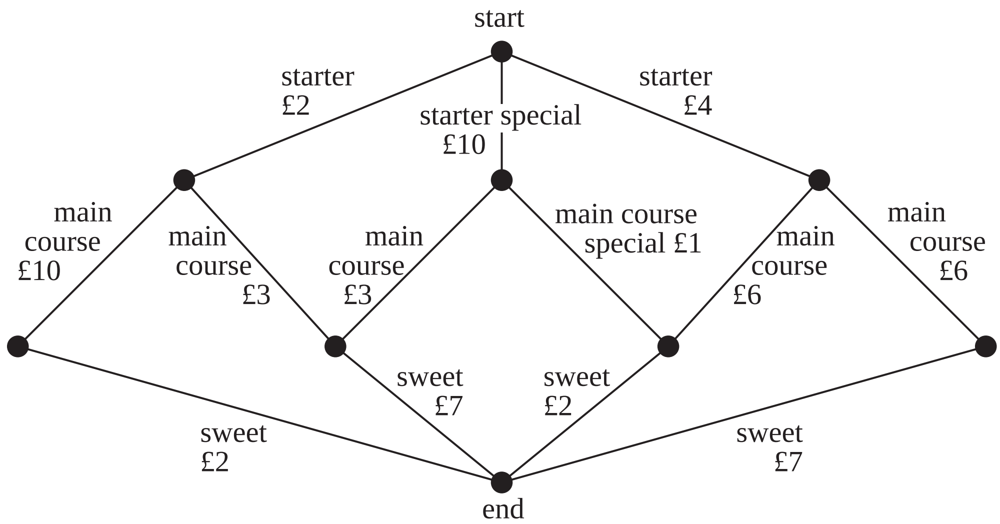
(i) Apply the algorithm when $n = 5$, showing the intermediate values of i, j and k. **[5]**

The function $f(n) = \frac{1}{6}n^3 - \frac{7}{6}n + 1$ gives the same output as the algorithm.

(ii) Showing your working, check that f(5) is the same value as your answer to part **(i)**. **[2]**

(iii) What does the structure of f(n) tell you about Prim's algorithm? **[1]**

3 Whilst waiting for her meal to be served, Alice tries to construct a network to represent the meals offered in the restaurant.



(i) Use Dijkstra's algorithm to find the cheapest route through the undirected network from "start" to "end". Give the cost and describe the route. Use the lettering given on the network in your answer book. **[6]**

(ii) Criticise the model and suggest how it might be improved. **[2]**

Section B (48 marks)

4 A ski-lift gondola can carry 4 people. The weight restriction sign in the gondola says "4 people – 325 kg".

The table models the distribution of weights of people using the gondola.

	Men	Women	Children
Weight (kg)	90	80	40
Probability	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

(i) Give an efficient rule for using 2-digit random numbers to simulate the weight of a person entering the gondola. **[5]**

(ii) Give a reason for using 2-digit rather than 1-digit random numbers in these circumstances. **[1]**

(iii) Using the random numbers given in your answer book, simulate the weights of four people entering the gondola, and hence give its simulated load. **[3]**

(iv) Using the random numbers given in your answer book, repeat your simulation 9 further times. Hence estimate the probability of the load of a fully-laden gondola exceeding 325 kg. **[6]**

(v) What in reality might affect the pattern of loading of a gondola which is not modelled by your simulation? **[1]**

5 The tasks involved in turning around an "AirGB" aircraft for its return flight are listed in the table. The table gives the durations of the tasks and their immediate predecessors.

Activity		Duration (mins)	Immediate Predecessors
A	Refuel	30	–
B	Clean cabin	25	–
C	Pre-flight technical check	15	A
D	Load luggage	20	–
E	Load passengers	25	A, B
F	Safety demonstration	5	E
G	Obtain air traffic clearance	10	C
H	Taxi to runway	5	G, D

(i) Draw an activity on arc network for these activities. **[4]**

(ii) Mark on your diagram the early time and the late time for each event. Give the minimum completion time and the critical activities. **[6]**

Because of delays on the outbound flight the aircraft has to be turned around within 50 minutes, so as not to lose its air traffic slot for the return journey. There are four tasks on which time can be saved. These, together with associated costs, are listed below.

Task	A	B	D	E
New time (mins)	20	20	15	15
Extra cost	250	50	50	100

(iii) List the activities which need to be speeded up in order to turn the aircraft around within 50 minutes at minimum extra cost. Give the extra cost and the new set of critical activities. **[6]**

6 A company is planning its production of "MPowder" for the next three months.

- Over the next 3 months 20 tonnes must be produced.
- Production quantities must not be decreasing. The amount produced in month 2 cannot be less than the amount produced in month 1, and the amount produced in month 3 cannot be less than the amount produced in month 2.
- No more than 12 tonnes can be produced in total in months 1 and 2.
- Production costs are £2000 per tonne in month 1, £2200 per tonne in month 2 and £2500 per tonne in month 3.

The company planner starts to formulate an LP to find a production plan which minimises the cost of production:

Minimise $2000x_1 + 2200x_2 + 2500x_3$
subject to $x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0$
$x_1 + x_2 + x_3 = 20$
$x_1 \leq x_2$
• • •

(i) Explain what the variables x_1, x_2 and x_3 represent, and write down two more constraints to complete the formulation. **[4]**

(ii) Explain how the LP can be reformulated to:

Maximise $500x_1 + 300x_2$
subject to $x_1 \geq 0 \quad x_2 \geq 0$
$x_1 \leq x_2$
$x_1 + 2x_2 \leq 20$
$x_1 + x_2 \leq 12$ **[3]**

(iii) Use a graphical approach to solve the LP in part **(ii)**. Interpret your solution in terms of the company's production plan, and give the minimum cost. **[9]**

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RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) **4771**

Decision Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- Printed Answer Book
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 17 June 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Do **not** write in the bar codes.

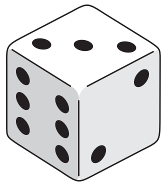
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Answer all questions in the printed answer book provided.

Section A (24 marks)

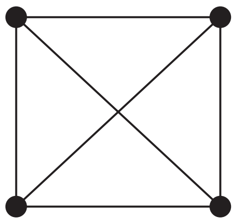
1 The numbers on opposite faces of the die shown (a standard die) add up to 7. The adjacency graph for the die is a graph which has vertices representing faces. In the adjacency graph two vertices are joined with an arc if they share an edge of the die. For example, vertices 2 and 6 are joined by an arc because they share an edge of the die.



(i) List the pairs of numbers which are opposite each other. **[1]**

(ii) Draw the adjacency graph. **[5]**

(iii) Identify and sketch a solid which has the following adjacency graph. **[2]**



2 In this question INT(m) means the integer part of m. Thus INT(3.5) = 3 and INT(4) = 4.

A game for two players starts with a number, n, of counters. Players alternately pick up a number of counters, **at least 1** and **not more than half** of those left. The player forced to pick up the last counter is the loser. Arif programs his computer to play the game, using the rule "pick up INT(half of the remaining counters), or the last counter if forced".

(i) You are to play against Arif's computer with $n = 5$ and with Arif's computer going first. What happens at each turn? **[3]**

(ii) You are to play against Arif's computer with $n = 6$ and with Arif's computer going first. What happens at each turn? **[2]**

(iii) Now play against Arif's computer with $n = 7$ and with Arif's computer going first. Describe what happens. **[3]**

3 Consider the following linear programming problem:

Maximise $3x + 4y$

subject to $2x + 5y \leqslant 60$
$x + 2y \leqslant 25$
$x + y \leqslant 18$

(i) Graph the inequalities and hence solve the LP. **[6]**

(ii) The right-hand side of the second inequality is increased from 25. At what new value will this inequality become redundant? **[2]**

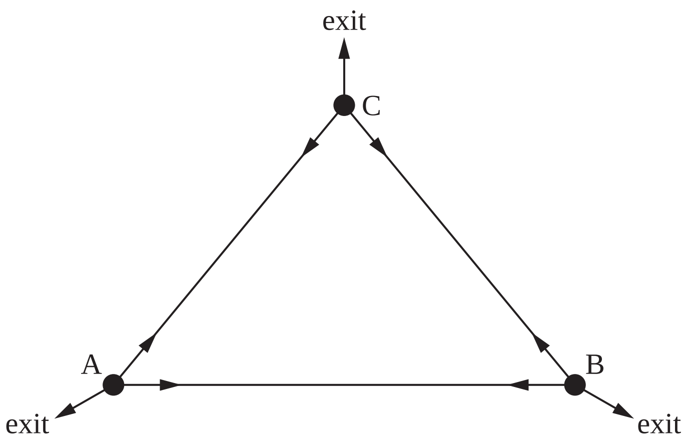
Section B (48 marks)

4 The diagram represents a very simple maze with two vertices, A and B. At each vertex a rat either exits the maze or runs to the other vertex, each with probability 0.5. The rat starts at vertex A.

(i) Describe how to use 1-digit random numbers to simulate this situation. **[2]**

(ii) Use the random digits provided in your answer book to run 10 simulations, each starting at vertex A. Hence estimate the probability of the rat exiting at each vertex, and calculate the mean number of times it runs between vertices before exiting. **[5]**

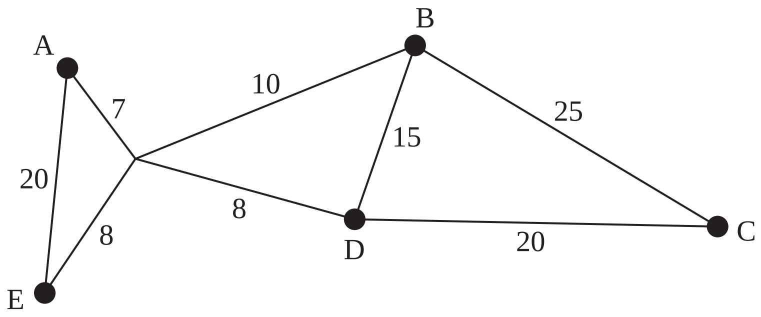
The second diagram represents a maze with three vertices, A, B and C. At each vertex there are three possibilities, and the rat chooses one, each with probability 1/3. The rat starts at vertex A.



(iii) Describe how to use 1-digit random numbers to simulate this situation. **[4]**

(iv) Use the random digits provided in your answer book to run 10 simulations, each starting at vertex A. Hence estimate the probability of the rat exiting at each vertex. **[5]**

5 The diagram represents canals connecting five cities. Canal lengths (shown on the arcs) are in km.

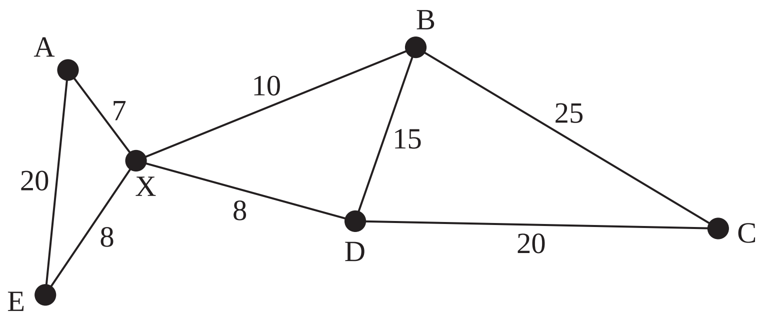


(i) Draw a network in your answer book with nodes representing the five cities and arcs representing direct canal connections, i.e. canal connections which do not involve passing through another city. **[3]**

The company operating the canal system wishes to close some canals to save money, whilst preserving the connectivity.

(ii) Starting at A, use Prim's algorithm on your answer to part **(i)** to find a minimum connector for the network. Give the order in which you include arcs. Draw your minimum connector and give its total length. **[5]**

Consider the original network together with an extra vertex, X, at the junction of four arcs.



(iii) Draw the minimum connector which results from applying Prim's algorithm, starting at A, to this network. Give the length of that minimum connector.
Hence advise the company on which canals to close. **[7]**

(iv) Give a reason why the company might face objections to such closures. **[1]**

Question 6 is printed overleaf.

6 Joan and Keith have to clear and tidy their garden. The table shows the jobs that have to be completed, their durations and their precedences.

Jobs		Duration (mins)	Immediate predecessors
A	prune bushes	100	–
B	weed borders	60	A
C	cut hedges	150	–
D	hoe vegetable patch	60	–
E	mow lawns	40	B
F	edge lawns	20	D, E
G	clean up cuttings	30	B, C
H	clean tools	10	F, G

(i) Draw an activity on arc network for these activities. **[4]**

(ii) Mark on your diagram the early time and the late time for each event. Give the minimum completion time and the critical activities. **[6]**

(iii) Each job is to be done by one person only. Joan and Keith are equally able to do all jobs. Draw a cascade chart indicating how to organise the jobs so that Joan and Keith can complete the project in the least time. Give that least time and explain why the minimum project completion time is shorter. **[6]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4771

Decision Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book

OCR Supplied Materials:
- Printed Answer Book 4771
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 25 January 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- **The questions are on the inserted Question Paper.**
- **Write your answer to each question in the space provided in the Printed Answer Book.** If you need more space for an answer use a 4-page answer book; label your answer clearly. Write your Centre Number and Candidate Number on the 4-page answer book and attach it securely to the Printed Answer Book.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

Answer all questions on the Printed Answer Book provided.

Section A (24 marks)

1 The table shows the activities involved in a project, their durations and their precedences.

Activity	Duration (mins)	Immediate predecessors
A	3	–
B	2	–
C	3	A
D	5	A, B
E	1	C

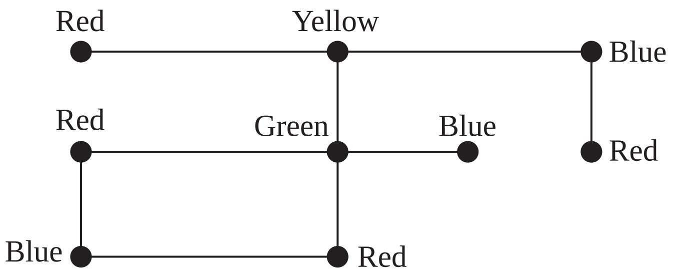
(i) Draw an activity on arc network for these activities. **[3]**

(ii) Mark on your diagram the early time and the late time for each event. Give the critical activities. **[5]**

2 The vertices of a graph are to be coloured using the following rules:

- all vertices are to be coloured
- no two vertices joined by an edge are to have the same colour.

The following graph has been coloured with four colours.



Kempe's rule allows for colours to be swapped. The rule is:

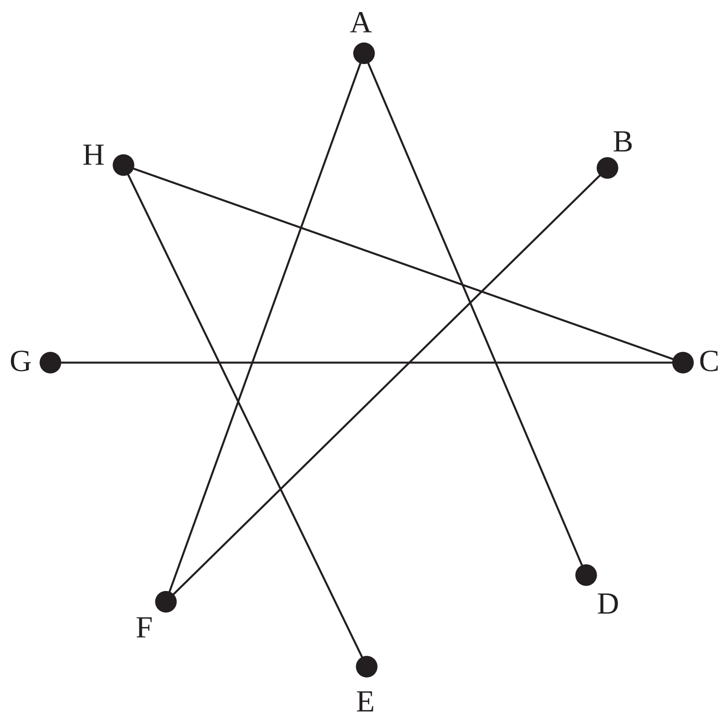
- choose two colours
- draw the subgraph consisting of the vertices coloured with these two colours, together with the edges that connect them
- in any connected part of this subgraph consisting of two or more vertices, the two colours can be swapped.

(i) Use Kempe's rule, choosing the colours blue and red.

Show that the graph can then be coloured with two colours. **[7]**

(ii) Why does Kempe's rule not constitute an algorithm for colouring graphs? **[1]**

3 Consider the following graph in which the arcs are straight lines.



(i) Explain how you know that the graph is simple. **[2]**

(ii) Explain how you know that the graph is not connected. **[2]**

(iii) On the copy of the graph in your answer book, add as many arcs as you can whilst keeping it both simple and not connected. Give the number of arcs which you have added. **[3]**

(iv) Imagine that a new graph is produced in which two vertices are connected if there is no connection between them, direct or indirect, on the original graph. How many arcs would this new graph have? **[1]**

Section B (48 marks)

4 An air charter company has the following rules for selling seats on a flight.

1. The total number of seats sold must not exceed 120.
2. There must be at least 100 seats sold, or the flight will be cancelled.
3. For every child seat sold there must be a seat sold for a supervising adult.

(i) Define two variables so that the three constraints can be formulated in terms of your variables. Formulate the three constraints in terms of your variables. **[5]**

(ii) Graph your three inequalities from part **(i)**. **[4]**

The price for a child seat is £50 and the price for an adult seat is £100.

(iii) Find the maximum income available from the flight, and mark and label the corresponding point on your graph. **[1]**

(iv) Find the minimum income available from a full plane, and mark and label the corresponding point on your graph. **[2]**

(v) Find the minimum income available from the flight, and mark and label the corresponding point on your graph. **[2]**

(vi) At £100 for an adult seat and £50 for a child seat the company would prefer to sell 100 adult seats and no child seats rather than have a full plane with 60 adults and 60 children. What would be the minimum price for a child's seat for that not to be the case, given that the adult seat price remains at £100? **[2]**

5 The matrix shows the distances in miles between towns where direct routes exist.

	A	B	C	D	E	F
A	–	22	–	12	10	–
B	22	–	–	–	–	13
C	–	–	–	6	5	11
D	12	–	6	–	–	–
E	10	–	5	–	–	26
F	–	13	11	–	26	–

(i) Draw the network. **[3]**

(ii) Use Dijkstra's algorithm to find the shortest route from A to F. Give the route and its length. **[6]**

(iii) Use Kruskal's algorithm to find a minimum connector for the network, showing your working. Draw your connector and give its total length. **[5]**

(iv) How much shorter would AD have to be if it were to be included in
(A) a shortest route from A to F,
(B) a minimum connector? **[2]**

[Question 6 is printed overleaf]

6 An apple tree has 6 apples left on it. Each day each remaining apple has a probability of $\frac{1}{3}$ of falling off the tree during the day.

(i) Give a rule for using one-digit random numbers to simulate whether or not a particular apple falls off the tree during a given day. **[3]**

(ii) Use the random digits given in your answer book to simulate how many apples fall off the tree during day 1. Give the total number of apples that fall during day 1. **[4]**

(iii) Continue your simulation from the end of day 1, which you simulated in part **(ii)**, for successive days until there are no apples left on the tree. Use the same list of random digits, continuing from where you left off in part **(ii)**.

During which day does the last apple fall from the tree? **[4]**

Now suppose that at the start of each day the gardener picks one apple from the tree and eats it.

(iv) Repeat your simulation with the gardener picking the lowest numbered apple remaining on the tree at the start of each day. Give the day during which the last apple falls or is picked.
Use the same string of random digits, a copy of which is provided for your use in this part of the question. **[4]**

(v) How could your results be made more reliable? **[1]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4771

Decision Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book

OCR Supplied Materials:
- Printed Answer Book 4771
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Tuesday 22 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- **The questions are on the inserted Question Paper.**
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

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- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

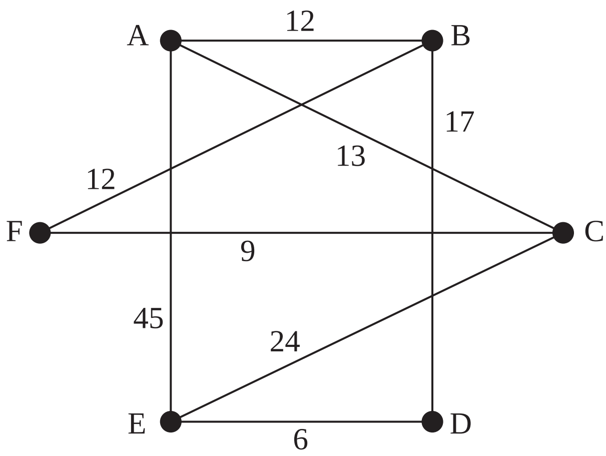
- Do not send this Question Paper for marking; it should be retained in the centre or destroyed.

Turn over

Answer all questions on the Printed Answer Book provided.

Section A (24 marks)

1 **(i)** Use Dijkstra's algorithm to find the shortest distances and corresponding routes from A to each of the other vertices in the given network. **[7]**



(ii) If the shortest distances and routes between every pair of vertices are required how many applications of Dijkstra will be needed? **[1]**

2 The following steps define an algorithm which acts on two numbers.

STEP 1 Write down the two numbers side by side on the same row.

STEP 2 Beneath the left-hand number write down double that number. Beneath the right-hand number write down half of that number, ignoring any remainder.

STEP 3 Repeat STEP 2 until the right-hand number is 1.

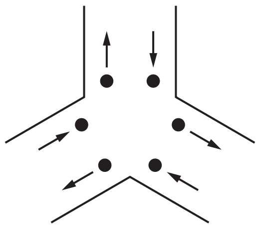
STEP 4 Delete those rows where the right-hand number is even. Add up the remaining left-hand numbers. This is the result.

(i) Apply the algorithm to the left-hand number 3 and the right-hand number 8. **[3]**

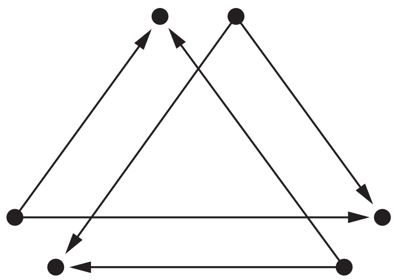
(ii) Apply the algorithm to the left-hand number 26 and the right-hand number 42. **[4]**

(iii) Use your results from parts **(i)** and **(ii)**, together with any other examples you may choose, to write down what the algorithm achieves. **[1]**

3 Traffic flows in and out of a junction of three roads as shown in the diagram.

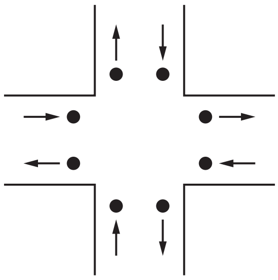


Assuming that no traffic leaves the junction by the same road as it entered, then the digraph shows traffic flows across the junction.



(i) Redraw the digraph to show that it is planar. **[3]**

(ii) Draw a digraph to show the traffic flow across the junction of 4 roads, assuming that no traffic leaves the junction by the same road as it entered.



[4]

(Note that the resulting digraph is also planar, but you are not required to show this.)

(iii) The digraphs showing flows across the junctions omit an important aspect in their modelling of the road junctions. What is it that they omit? **[1]**

Section B (48 marks)

4 A wall 4 metres long and 3 metres high is to be tiled. Two sizes of tile are available, 10 cm by 10 cm and 30 cm by 30 cm.

(i) If x is the number of boxes of **ten** small tiles used, and y is the number of large tiles used, explain why $10x + 9y \geqslant 1200$. **[3]**

There are only 100 of the large tiles available.
The tiler advises that the area tiled with the small tiles should not exceed the area tiled with the large tiles.

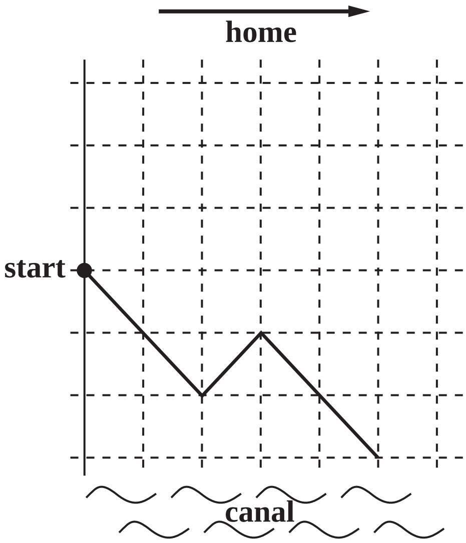
(ii) Express these two constraints in terms of x and y. **[3]**

The smaller tiles cost 15p each and the larger tiles cost £2 each.

(iii) Given that the objective is to minimise the cost of tiling the wall, state the objective function. Use the graphical approach to solve the problem. **[8]**

(iv) Give **two** other factors which would have to be taken into account in deciding how many of each tile to purchase. **[2]**

5 The diagram shows the progress of a drunkard towards his home on one particular night. For every step which he takes towards his home, he staggers one step diagonally to his left or one step diagonally to his right, randomly and with equal probability. There is a canal three steps to the right of his starting point, and no constraint to the left. On this particular occasion he falls into the canal after 5 steps.



(i) Explain how you would simulate the drunkard's walk, making efficient use of one-digit random numbers. **[3]**

(ii) Using the random digits in the Printed Answer Book simulate the drunkard's walk and show his progress on the grid. Stop your simulation either when he falls into the canal or when he has staggered 6 steps, whichever happens first. **[2]**

(iii) How could you estimate the probability of him falling into the canal within 6 steps? **[2]**

On another occasion the drunkard sets off carrying a briefcase in his right hand. This changes the probabilities of him staggering to the right to $\frac{2}{3}$, and to the left to $\frac{1}{3}$.

(iv) Explain how you would now simulate this situation. **[3]**

(v) Simulate the drunkard's walk (with briefcase) 10 times, and hence estimate the probability of him falling into the canal within 6 steps. (In your simulations you are not required to show his progress on a grid. You only need to record his steps to the right or left.) **[6]**

Question 6 is printed overleaf.

6 The table shows the tasks that have to be completed in building a stadium for a sporting event, their durations and their precedences. The stadium has to be ready within two years.

Task	Duration (months)	Immediate predecessors
A	4	–
B	2	–
C	7	–
D	12	A
E	5	A
F	7	A, B
G	6	D, J
H	3	C
I	12	E, F, H
J	7	E, F, H
K	12	C

(i) Draw an activity on arc network for these activities. **[5]**

(ii) Mark on your diagram the early time and the late time for each event. Give the project duration and the critical activities. **[6]**

In the later stages of planning the project it is discovered that task J will actually take 9 months to complete. However, other tasks can have their durations shortened by employing extra resources. The costs of "crashing" tasks (i.e. the costs of employing extra resources to complete them more quickly) are given in the table.

Tasks which can be completed more quickly by employing extra resources	Number of months which can be saved	Cost per month of employing extra resources (£m)
A	2	3
D	1	1
C	3	3
F	2	2
G	2	4

(iii) Find the cheapest way of completing the project within two years. **[3]**

(iv) If the delay in completing task J is not discovered until it is started, how can the project be completed in time, and how much extra will it cost? **[2]**

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RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4771

Decision Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4771
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 24 January 2011
Morning

Duration: 1 hour 30 minutes

OCE/18047

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
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INFORMATION FOR CANDIDATES

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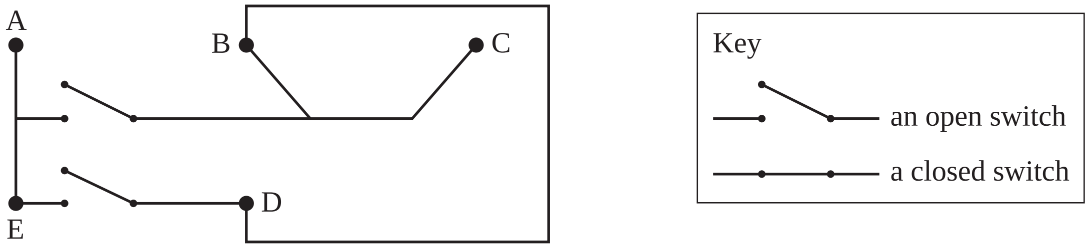
INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Answer all questions in the Printed Answer Book provided.

Section A (24 marks)

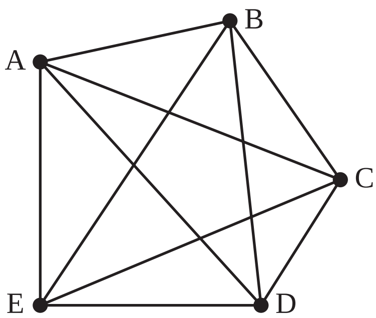
1 The diagram shows an electrical circuit with wires and switches and with five components, labelled A, B, C, D and E.



(i) Draw a graph showing which vertices are connected together, either directly or indirectly, when the two switches remain open. **[2]**

(ii) How many arcs need to be added to your graph when both switches are closed? **[2]**

The graph below shows which components are connected to each other, either directly or indirectly, for a second electrical circuit.



(iii) Find the minimum number of arcs which need to be deleted to create two disconnected sets of vertices, and write down your two separate sets. **[3]**

(iv) Explain why, in the second electrical circuit, it might be possible to split the components into two disconnected sets by cutting fewer wires than the number of arcs which were deleted in part **(iii)**. **[1]**

2 King Elyias has been presented with eight flagons of fine wine. Intelligence reports indicate that at least one of the eight flagons has been poisoned. King Elyias will have the wine tasted by the royal wine tasters to establish which flagons are poisoned.

Samples for testing are made by using wine from one or more flagons. If a royal wine taster tastes a sample of wine which includes wine from a poisoned flagon, the taster will die. The king has to make a very generous payment for each sample tasted.

To minimise payments, the royal mathematicians have devised the following scheme:

Test a sample made by mixing wine from flagons 1, 2, 3 and 4.

If the taster dies, then test a sample made by mixing wine from flagons 5, 6, 7 and 8.

If the taster lives, then there is no poison in flagons 1, 2, 3 or 4. So there **is** poison in at least one of flagons 5, 6, 7 and 8, and there is no need to test a sample made by mixing wine from all four of them.

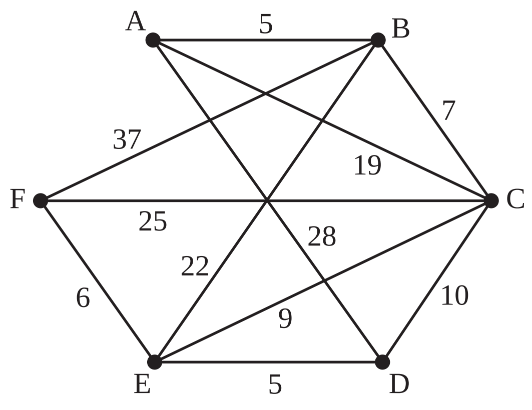
If the sample from flagons 1, 2, 3 and 4 contains poison, then test a fresh sample made by mixing wine from flagons 1 and 2, and proceed similarly, testing a sample from flagons 3 and 4 only if the taster of the sample from flagons 1 and 2 dies.

Continue, testing new samples made from wine drawn from half of the flagons corresponding to a poisoned sample, and testing only when necessary.

(i) Record what happens using the mathematicians' scheme when flagon number 7 is poisoned, and no others. **[4]**

(ii) Record what happens using the mathematicians' scheme when two flagons, numbers 3 and 7, are poisoned. **[4]**

3 The network shows distances between vertices where direct connections exist.



(i) Use Dijkstra's algorithm to find the shortest distance and route from A to F. **[6]**

(ii) Explain why your solution to part **(i)** also provides the shortest distances and routes from A to each of the other vertices. **[1]**

(iii) Explain why your solution to part **(i)** also provides the shortest distance and route from B to F. **[1]**

Section B (48 marks)

4 The table shows the tasks involved in preparing breakfast, and their durations.

Task	Description	Duration (mins)
A	Fill kettle and switch on	0.5
B	Boil kettle	1.5
C	Cut bread and put in toaster	0.5
D	Toast bread	2
E	Put eggs in pan of water and light gas	1
F	Boil eggs	5
G	Put tablecloth, cutlery and crockery on table	2.5
H	Make tea and put on table	0.5
I	Collect toast and put on table	0.5
J	Put eggs in cups and put on table	1

(i) Show the immediate predecessors for each of these tasks. **[3]**

(ii) Draw an activity on arc network modelling your precedences. **[3]**

(iii) Perform a forward pass and a backward pass to find the early time and the late time for each event. **[4]**

(iv) Give the critical activities, the project duration, and the total float for each activity. **[3]**

(v) Given that only one person is available to do these tasks, and noting that tasks B, D and F do not require that person's attention, produce a cascade chart showing how breakfast can be prepared in the least possible time. **[3]**

5 Viola and Orsino are arguing about which striker to include in their fantasy football team. Viola prefers Rocinate, who creates lots of goal chances, but is less good at converting them into goals. Orsino prefers Quince, who is not so good at creating goal chances, but who is better at converting them into goals.

The information for Rocinate and Quince is shown in the tables.

	Number of chances created per match								
	Rocinate					Quince			
Number	6	7	8	9		5	6	7	8
Probability	$\frac{1}{20}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{5}$		$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

Probability of converting a chance into a goal	
Rocinate	Quince
0.1	0.12

(i) Give an efficient rule for using 2-digit random numbers to simulate the number of chances created by Rocinate in a match. **[3]**

(ii) Give a rule for using 2-digit random numbers to simulate the conversion of chances into goals by Rocinate. **[1]**

(iii) Your Printed Answer Book shows the result of simulating the number of goals scored by Rocinate in nine matches. Use the random numbers given to complete the tenth simulation, showing which of your simulated chances are converted into goals. **[3]**

(iv) Give an efficient rule for using 2-digit random numbers to simulate the number of chances created by Quince in a match. **[3]**

(v) Your Printed Answer Book shows the result of simulating the number of goals scored by Quince in nine matches. Use the random numbers given to complete the tenth simulation, showing which of your simulated chances are converted into goals. **[3]**

(vi) Which striker, if any, is favoured by the simulation? Justify your answer. **[2]**

(vii) How could the reliability of the simulation be improved? **[1]**

[Question 6 is printed overleaf.]

6 A manufacturing company holds stocks of two liquid chemicals. The company needs to update its stock levels.

The company has 2000 litres of chemical A and 4000 litres of chemical B currently in stock. Its storage facility allows for no more than a combined total of 12 000 litres of the two chemicals.

Chemical A is valued at £5 per litre and chemical B is valued at £6 per litre. The company intends to hold stocks of these two chemicals with a total value of at least £61 000.

Let a be the increase in the stock level of A, in thousands of litres (a can be negative).
Let b be the increase in the stock level of B, in thousands of litres (b can be negative).

(i) Explain why $a \geqslant -2$, and produce a similar inequality for b. **[2]**

(ii) Explain why the value constraint can be written as $5a + 6b \geqslant 27$, and produce, in similar form, the storage constraint. **[4]**

(iii) Illustrate all four inequalities graphically. **[5]**

(iv) Find the policy which will give a stock value of exactly £61 000, and will use all 12 000 litres of available storage space. **[2]**

(v) Interpret your solution in terms of stock levels, and verify that the new stock levels do satisfy both the value constraint and the storage constraint. **[3]**

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4771

Decision Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4771
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 22 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

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- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
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INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

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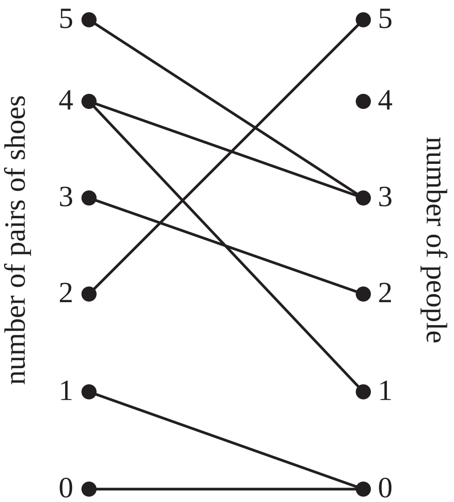
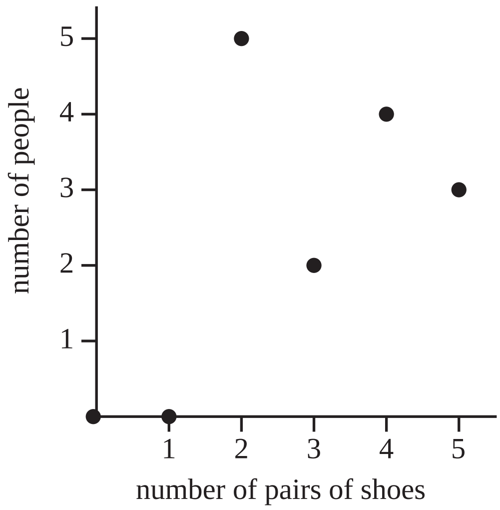
Turn over

Section A (24 marks)

1 Two students draw graphs to represent the numbers of pairs of shoes owned by members of their class. Andrew produces a bipartite graph, but gets it wrong. Barbara produces a completely correct frequency graph. Their graphs are shown below.



Andrew's graph



Barbara's graph

(i) Draw a correct bipartite graph. **[3]**

(ii) How many people are in the class? **[1]**

(iii) How many pairs of shoes in total are owned by members of the class? **[2]**

(iv) Which points on Barbara's graph may be deleted without losing any information? **[1]**

Charles produces the same frequency graph as Barbara, but joins consecutive points with straight lines.

(v) Criticise Charles's graph. **[1]**

2 The algorithm gives a method for drawing two straight lines, if certain conditions are met.

> Start with the equations of the two straight lines
>
> Line 1 is $ax + by = c, \quad a, b, c > 0$
> Line 2 is $dx + ey = f, \quad d, e, f > 0$
>
> Let X = minimum of $\frac{c}{a}$ and $\frac{f}{d}$
> Let Y = minimum of $\frac{c}{b}$ and $\frac{f}{e}$
>
> If $X = \frac{c}{a}$ then $X^* = \frac{c - bY}{a}$ and $Y^* = \frac{f - dX}{e}$
>
> If $X = \frac{f}{d}$ then $X^* = \frac{f - eY}{d}$ and $Y^* = \frac{c - aX}{b}$
>
> Draw an x-axis labelled from 0 to X, and a y-axis labelled from 0 to Y
> Join $(0, Y)$ to (X, Y^*) with a straight line
> Join (X^*, Y) to $(X, 0)$ with a straight line

(i) Apply the algorithm with $a = 1$, $b = 5$, $c = 25$, $d = 10$, $e = 2$, $f = 85$. **[7]**

(ii) Why might this algorithm be useful in an LP question? **[1]**

3 John has a standard die in his pocket (ie a cube with its six faces labelled from 1 to 6).

(i) Describe how John can use the die to obtain realisations of the random variable X, defined below.

x	1	2	3
Probability($X = x$)	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$

[3]

(ii) Describe how John can use the die to obtain realisations of the random variable Y, defined below.

y	1	2	3
Probability($Y = y$)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

[3]

(iii) John attempts to use the die to obtain a realisation of a uniformly distributed 2-digit random number. He throws the die 20 times. Each time he records one less than the number showing. He then adds together his 20 recorded numbers.

Criticise John's methodology. **[2]**

Section B (48 marks)

4 An eco-village is to be constructed consisting of large houses and standard houses.

Each large house has 4 bedrooms, needs a plot size of 200 m^2 and costs £60 000 to build.

Each standard house has 3 bedrooms, needs a plot size of 120 m^2 and costs £50 000 to build.

The area of land available for houses is 120 000 m^2. The project has been allocated a construction budget of £42.4 million.

The market will not sustain more than half as many large houses as standard houses. So, for instance, if there are 500 standard houses then there must be no more than 250 large houses.

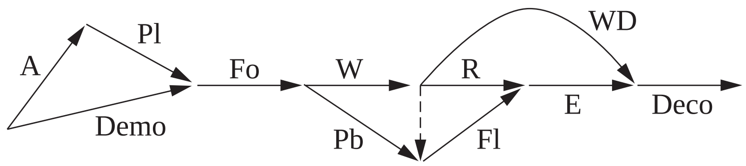
(i) Define two variables so that the three constraints can be formulated in terms of your variables. Formulate the three constraints in terms of your variables. **[5]**

(ii) Graph your three inequalities from part **(i)**, indicating the feasible region. **[4]**

(iii) Find the maximum number of bedrooms which can be provided, and the corresponding numbers of each type of house. **[2]**

(iv) Modify your solution if the construction budget is increased to £45 million. **[5]**

5 The activity network and table together show the tasks involved in constructing a house extension, their durations and precedences.



Activity	Description	Duration (days)
A	Architect produces plans	10
Pl	Obtain planning permission	14
Demo	Demolish existing structure	3
Fo	Excavate foundations	4
W	Build walls	3
Pb	Install plumbing	2
R	Construct roof	3
Fl	Lay floor	2
E	Fit electrics	2
WD	Install windows and doors	1
Deco	Decorate	5

(i) Show the immediate predecessors for each activity. **[2]**

(ii) Perform a forward pass and a backward pass to find the early time and the late time for each event. **[4]**

(iii) Give the critical activities, the project duration, and the total float for each activity. **[4]**

(iv) The activity network includes one dummy activity. Explain why this dummy activity is needed. **[2]**

Whilst the foundations are being dug the customer negotiates the installation of a decorative corbel. This will take one day. It must be done after the walls have been built, and before the roof is constructed. The windows and doors cannot be installed until it is completed. It will not have any effect on the construction of the floor.

(v) Redraw the activity network incorporating this extra activity. **[3]**

(vi) Find the revised critical activities and the revised project duration. **[1]**

6 The table shows the distances in miles, where direct rail connections are possible, between 11 cities in a country. The government is proposing to construct a high-speed rail network to connect the cities.

	P	S	F	Ln	Br	Nr	Bm	Ld	Nc	Lv	M
P	–	150	–	240	125	–	–	–	–	–	–
S	150	–	150	80	105	–	135	–	–	–	–
F	–	150	–	80	–	–	–	–	–	–	–
Ln	240	80	80	–	120	115	120	–	–	–	–
Br	125	105	–	120	–	230	90	–	–	–	–
Nr	–	–	–	115	230	–	160	175	255	–	–
Bm	–	135	–	120	90	160	–	120	–	–	90
Ld	–	–	–	–	–	175	120	–	210	100	90
Nc	–	–	–	–	–	255	–	210	–	175	–
Lv	–	–	–	–	–	–	–	100	175	–	35
M	–	–	–	–	–	–	90	90	–	35	–

(i) Use the tabular form of Prim's algorithm, starting at vertex P, to find a minimum connector for the network. Draw your minimum connector and give its total length. **[6]**

(ii) Give one advantage and two disadvantages of constructing a rail network using only the arcs of a minimum connector. **[3]**

(iii) Use Dijkstra's algorithm on the diagram in the Printed Answer Book, to find the shortest route and distance from P to Nr in the original network. **[6]**

(iv) Give the shortest distance from P to Nr using only arcs in your minimum connector. **[1]**

OCR
RECOGNISING ACHIEVEMENT

Monday 23 January 2012 – Morning

AS GCE MATHEMATICS (MEI)

4771 Decision Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4771
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4732210112

INSTRUCTIONS TO CANDIDATES

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- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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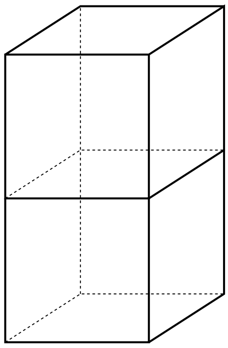
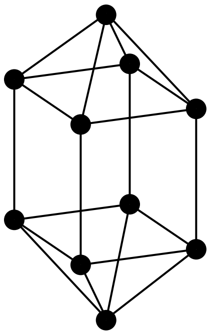
Section A (24 marks)

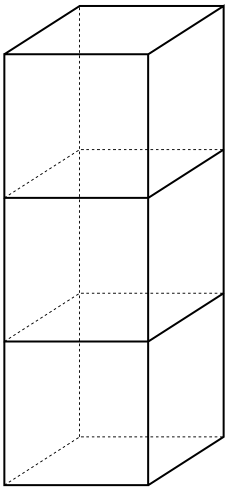
1 A graph is obtained from a solid by producing a vertex for each exterior face. Vertices in the graph are connected if their corresponding faces in the original solid share an edge. The diagram shows a solid followed by its graph. The solid is made up of two cubes stacked one on top of the other. This solid has 10 exterior faces, which correspond to the 10 vertices in the graph. (Note that in this question it is the exterior faces of the cubes that are being counted.)





(i) Draw the graph for a cube. **[4]**

(ii) Obtain the number of vertices and the number of edges for the graph of three cubes stacked on top of each other. **[4]**



2 The following is called the ‘1089’ algorithm. In steps 1 to 4 numbers are to be written with exactly three digits; for example 42 is written as 042.

Step 1 Choose a 3-digit number, with no digit being repeated.

Step 2 Form a new number by reversing the order of the three digits.

Step 3 Subtract the smaller number from the larger and call the difference D. If the two numbers are the same then D = 000.

Step 4 Form a new number by reversing the order of the three digits of D, and call it R.

Step 5 Find the sum of D and R.

(i) Apply the algorithm, choosing 427 for your 3-digit number, and showing all of the steps. **[4]**

(ii) Apply the algorithm to a 3-digit number of your choice, showing all of the steps. **[2]**

(iii) Investigate what happens if digits may be repeated in the 3-digit number in step 1. **[2]**

3 Solve the following LP problem graphically.

Maximise $2x + 3y$

subject to $x + y \leqslant 11$

$3x + 5y \leqslant 39$

$x + 6y \leqslant 39.$ **[8]**

Section B (48 marks)

4 The table defines a network in which the numbers represent lengths.

	A	B	C	D	E	F	G
A	–	5	2	3	–	–	–
B	5	–	–	–	1	1	–
C	2	–	–	–	4	1	–
D	3	–	–	–	4	2	–
E	–	1	4	4	–	–	1
F	–	1	1	2	–	–	5
G	–	–	–	–	1	5	–

(i) Draw the network. **[3]**

(ii) Use Dijkstra's algorithm to find the shortest paths from A to each of the other vertices. Give the paths and their lengths. **[6]**

(iii) Draw a new network containing all of the edges in your shortest paths, and find the total length of the edges in this network. **[2]**

(iv) Find a minimum connector for the original network, draw it, and give the total length of its edges. **[4]**

(v) Explain why the method defined by parts **(i)**, **(ii)** and **(iii)** does not always give a minimum connector. **[1]**

5 Five gifts are to be distributed among five people, A, B, C, D and E. The gifts are labelled from 1 to 5. Each gift is allocated randomly to one of the five people. A person can receive more than one gift.

(i) Use one-digit random numbers to simulate this process. One-digit random numbers are provided in your answer book.

Explain how your simulation works.

Produce a table, showing how many gifts each person receives. **[6]**

(ii) Carry out four more simulations showing, in each case, how many gifts each person receives. **[2]**

(iii) Use your simulation to estimate the probabilities of a person receiving 0, 1, 2, 3, 4 and 5 gifts. **[5]**

(iv) Describe what you would have to do differently if there were six people and six gifts. **[3]**

6 The table shows the tasks involved in making a salad, their durations and their precedences.

Task		Duration (seconds)	Immediate predecessors
B	get out bowl and implements	10	–
I	get out ingredients	10	–
L	chop lettuce	15	B, I
W	wash tomatoes and celery	25	B, I
T	chop tomatoes	15	W
C	chop celery	10	W
P	peel apple	20	B, I
A	chop apple	10	P
D	dress salad	10	L, T, C, A

(i) Draw an activity on arc network for these activities. **[5]**

(ii) Mark on your diagram the early and late times for each event. Give the minimum completion time and the critical activities. **[6]**

(iii) Given that each task can only be done by one person, how many people are needed to prepare the salad in the minimum time?

What is the minimum time required to prepare the salad if only one person is available? **[2]**

(iv) Show how two people can prepare the salad as quickly as possible. **[3]**

Thursday 31 May 2012 – Morning

AS GCE MATHEMATICS (MEI)

4771 Decision Mathematics 1

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:
- Question Paper 4771 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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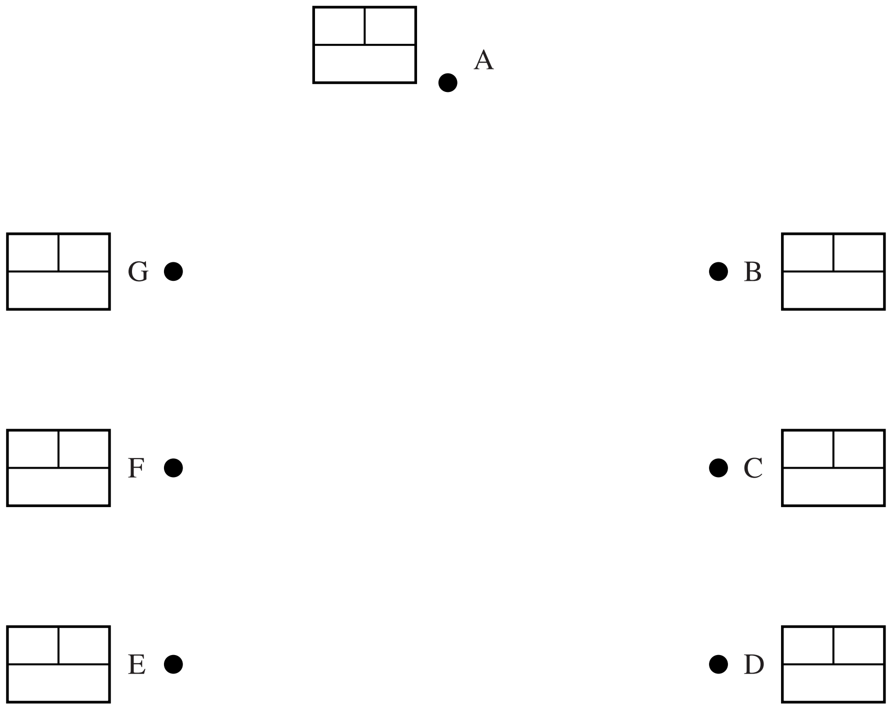
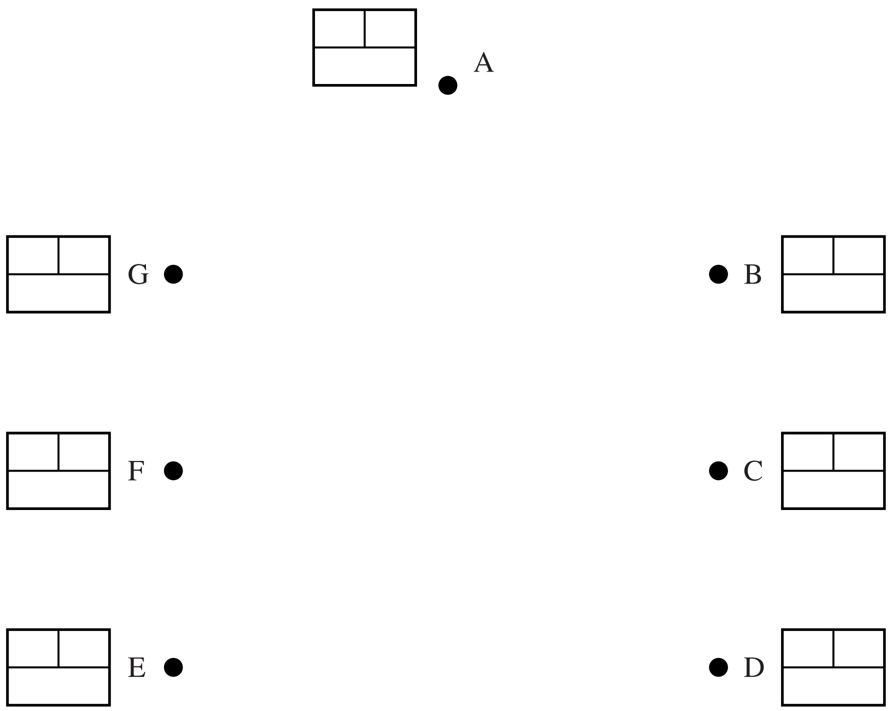
Section A (24 marks)

1 (i) & (ii)



Spare copy of diagram for 1(i)



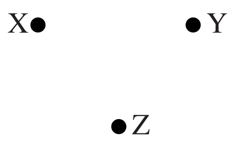
2 (i)

A	L	R	B	f(L)	f(R)

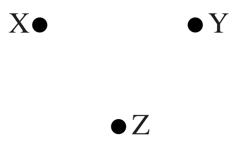
2 (ii)

2 (iii)

3 (i)



‘is a subset of’



‘shares at least one element with’

3 (ii)

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Section B (48 marks)

4 (i)

4 (ii)

A spare copy of this graph paper can be found on page 12.

4 (iii)	
4 (iv)	

PLEASE DO NOT WRITE IN THIS SPACE

5 (i)	
5 (ii)	
5 (iii)	Random digits: 5, 9, 4, 0, 8, 1, 9, 3, 0, 5
5 (iv)	Random digits for run 2: 0, 1, 3, 7, 5, 9, 9, 8, 7, 3
	Random digits for run 3: 9, 8, 4, 4, 2, 7, 7, 4, 1, 9
	(answer space continued on next page)

5 (iv) **(continued)**

Random digits for run 4: 9, 0, 5, 1, 6, 9, 6, 0, 6, 5

Random digits for run 5: 0, 9, 1, 8, 8, 9, 0, 6, 5, 5

5 (v)

Turn over

6 (i) & (ii)	

6 (iii)	
6 (iv)	
6 (v)	
6 (vi)	

Spare copy of graph paper for question 4(ii)

OCR
RECOGNISING ACHIEVEMENT

Friday 25 January 2013 – Afternoon

AS GCE MATHEMATICS (MEI)

4771/01 Decision Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4771/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO EXAMS OFFICER / INVIGILATOR

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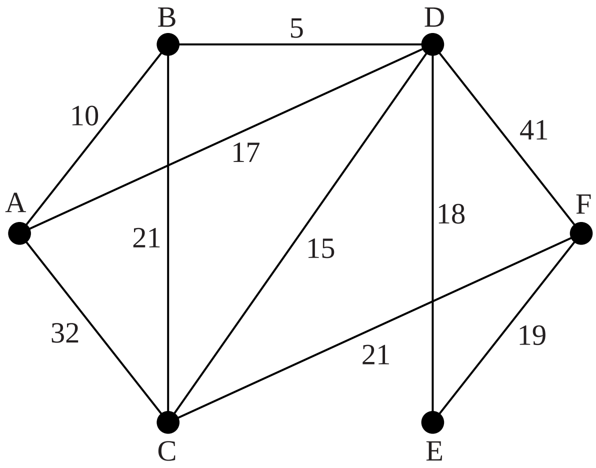
This paper has been pre modified for carrier language

Turn over

Section A (24 marks)

1 The weights on the arcs in the network represent times in minutes to travel between vertices.



(i) Use Dijkstra's algorithm to find the fastest route from A to F. Give the route and the time. **[5]**

(ii) Use an algorithm to find the minimum connector for the network, showing your working. Find the minimum time to travel from A to F using only arcs in the minimum connector. **[3]**

2 A small party is held in a country house. There are 10 men and 10 women, and there are 10 dances. For each dance a number of pairings, each of one man and one woman, are formed. The same pairing can appear in more than one dance. A graph is to be drawn showing who danced with whom during the evening, ignoring repetitions.

(i) Name the type of graph which is appropriate. **[1]**

(ii) What is the maximum possible number of arcs in the graph? **[2]**

Dashing Mr Darcy dances with every woman except Elizabeth, who will have nothing to do with him. She dances with eight different men.

Prince Charming only dances with Cinderella. Cinderella only dances with Prince Charming and with Mr Darcy.

The three ugly sisters only have one dance each.

(iii) Add arcs to the graph in your answer book to show this information. **[3]**

(iv) What is the maximum possible number of arcs in the graph? **[2]**

3 The following algorithm computes an estimate of the square root of a number which is between 0 and 2.

Step 1	Subtract 1 from the number and call the result x
Step 2	Set $oldr = 1$
Step 3	Set $i = 1$
Step 4	Set $j = 0.5$
Step 5	Set $k = 0.5$
Step 6	Set $change = x^i \times k$
Step 7	Set $newr = oldr + change$
Step 8	If $-0.005 < change < 0.005$ then go to Step 17
Step 9	Set $oldr = newr$
Step 10	Set $i = i + 1$
Step 11	Set $j = j - 1$
Step 12	Set $k = k \times j \div i$
Step 13	Set $change = x^i \times k$
Step 14	Set $newr = oldr + change$
Step 15	If $-0.005 < change < 0.005$ then go to Step 17
Step 16	Go to Step 9
Step 17	Print out $newr$

(i) Use the algorithm to find an estimate of the square root of 1.44, showing all of the steps. **[6]**

(ii) Consider what happens if the algorithm is applied to 0.56, and then use your four values of *change* from part **(i)** to calculate an estimate of the square root of 0.56. **[2]**

Section B (48 marks)

4 A room has two windows which have the same height but different widths. Each window is to have one curtain. The table lists the tasks involved in making the two curtains, their durations, and their immediate predecessors. The durations assume that only one person is working on the activity.

Task		Duration (minutes)	Immediate predecessor(s)
A	measure windows	5	–
B	calculate material required	5	A
C	choose material	15	–
D	buy material	15	B, C
E	cut material	5	D
F	stitch sides of wide curtain	30	E
G	stitch top of wide curtain	30	F
H	stitch sides of narrow curtain	30	E
I	stitch top of narrow curtain	15	H
J	hang curtains and pin hems	20	G, I
K	hem wide curtain	30	J
L	hem narrow curtain	15	J
M	fit curtains	10	K, L

(i) Draw an activity on arc network for these activities. **[5]**

(ii) Mark on your diagram the early time and the late time for each event. Give the minimum completion time and the critical activities. **[6]**

Kate and Pete have two rooms to curtain, each identical to that above. Tasks A, B, C and D only need to be completed once each. All other tasks will have two versions, one for room 1 and one for room 2, eg E1 and E2. Kate and Pete share the tasks between them so that each task is completed by only one person.

(iii) Complete the diagram to show how the tasks can be shared between them, and scheduled, so that the project can be completed in the least possible time. Give that least possible time. **[3]**

(iv) How much extra help would be needed to curtain both rooms in the minimum completion time from part **(ii)**? Explain your answer. **[2]**

5 A chairlift for a ski slope has 160 4-person chairs. At any one time half of the chairs are going up and half are coming down empty. An observer watches the loading of the chairs during a moderately busy period, and concludes that the number of occupants per 'up' chair has the following probability distribution.

number of occupants	0	1	2	3	4
probability	0.1	0.2	0.3	0.2	0.2

(i) Give a rule for using 1-digit random numbers to simulate the number of occupants of an up chair in a moderately busy period. **[2]**

(ii) Use the 10 random digits provided to simulate the number of occupants in 10 up chairs. **[2]**

The observer estimates that, at all times, on average 20% of chairlift users are children.

(iii) Give an efficient rule for using 1-digit random numbers to simulate whether an occupant of an up chair is a child or an adult. **[1]**

(iv) Use the random digits provided to simulate how many of the occupants of the 10 up chairs are children, and how many are adults. **There are more random digits than you will need**. **[2]**

(v) Use your results from part **(iv)** to estimate how many children and how many adults are on the chairlift (ie on the 80 up chairs) at any instant during a moderately busy period. **[1]**

In a very busy period the number of occupants of an up chair has the following probability distribution.

number of occupants	0	1	2	3	4
probability	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{3}{13}$	$\frac{3}{13}$	$\frac{5}{13}$

(vi) Give an efficient rule for using 2-digit random numbers to simulate the number of occupants of an up chair in a very busy period. **[3]**

(vii) Use the 2-digit random numbers provided to simulate the number of occupants in 5 up chairs. There are more random numbers provided than you will need. **[2]**

(viii) Simulate how many of the occupants of the 5 up chairs are children and how many are adults, and thus estimate how many children and how many adults are on the chairlift at any instant during a very busy period. **[2]**

(ix) Discuss the relative merits of simulating using a sample of 10 chairs as against simulating using a sample of 5 chairs. **[1]**

[Question 6 is printed overleaf.]

6 Jean knits items for charity. Each month the charity provides her with 75 balls of wool.

She knits hats and scarves. Hats require 1.5 balls of wool each and scarves require 3 balls each. Jean has 100 hours available each month for knitting. Hats require 4 hours each to make, and scarves require 2.5 hours each.

The charity sells the hats for £7 each and the scarves for £10 each, and wants to gain as much income as possible.

Jean prefers to knit hats but the charity wants no more than 20 per month. She refuses to knit more than 20 scarves each month.

(i) Define appropriate variables, construct inequality constraints, and draw a graph representing the feasible region for this decision problem. **[10]**

(ii) Give the objective function and find the integer solution which will give Jean's maximum monthly income. **[4]**

(iii) If the charity drops the price of hats in a sale to £4 each, what would be an optimal number of hats and scarves for Jean to knit? Assuming that all hats and scarves are sold, by how much would the monthly income drop? **[2]**

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RECOGNISING ACHIEVEMENT

Thursday 6 June 2013 – Morning

AS GCE MATHEMATICS (MEI)

4771/01 Decision Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4771/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

* 4 7 1 5 8 9 0 6 1 3 *

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INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

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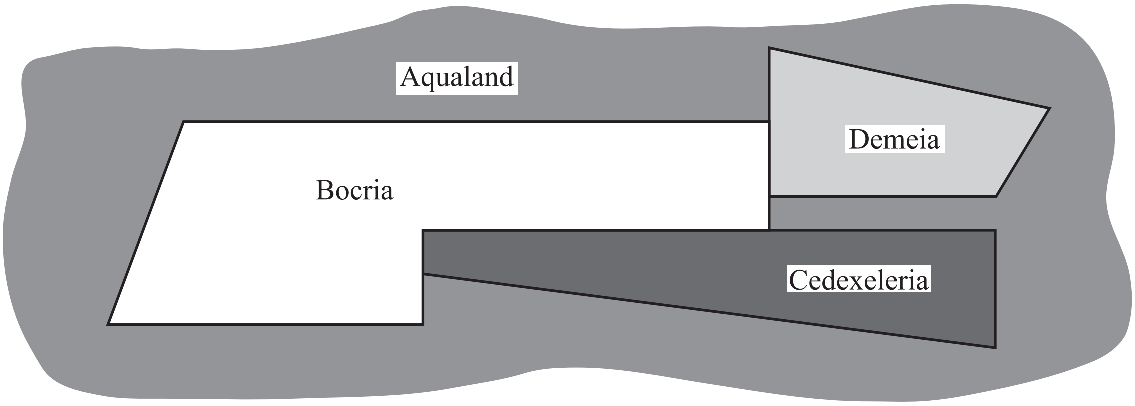
Section A (24 marks)

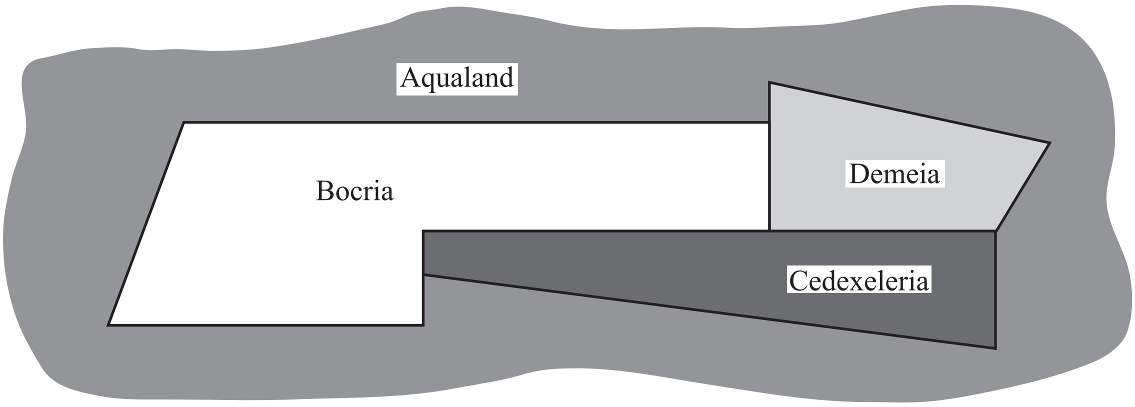
1 The adjacency graph for a map has a vertex for each country. Two vertices are connected by an arc if the corresponding countries share a border.

(i) Draw the adjacency graph for the following map of four countries. The graph is planar and you should draw it with no arcs crossing. **[3]**



(ii) Number the regions of your planar graph, including the outside region. Regarding the graph as a map, draw its adjacency graph. **[2]**

(iii) Repeat parts **(i)** and **(ii)** for the following map. **[3]**



2 The instructions labelled 1 to 7 describe the steps of a sorting algorithm applied to a list of six numbers.

```
1   Let i equal 1.
2   Repeat lines 3 to 7, stopping when i becomes 6.
3       Let j equal 1.
4       Repeat lines 5 and 6, until j becomes 7 – i.
5           If the jth number in the list is bigger than the (j+1)th, then swap them.
6           Let the new value of j be j+1.
7       Let the new value of i be i+1.
```

(i) Apply the sorting algorithm to the list of numbers shown below. Record in the table provided the state of the list after each pass. Record the number of comparisons and the number of swaps that you make in each pass. (The result of the first pass has already been recorded.) **[5]**

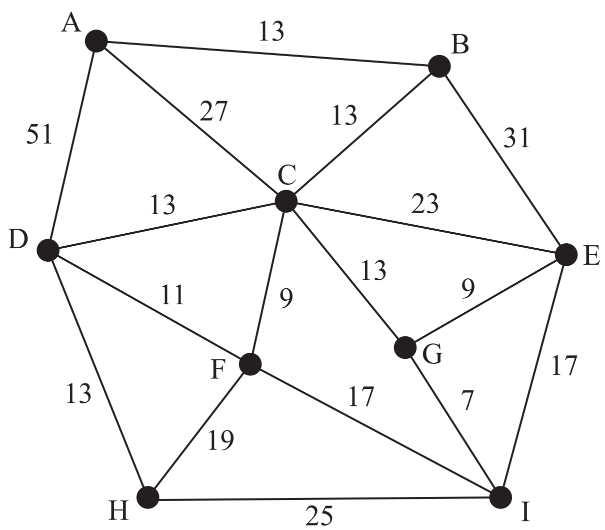
List: 9 11 7 3 13 5

(ii) Suppose now that the list is split into two sublists, {9, 11, 7} and {3, 13, 5}. The sorting algorithm is adapted to apply to three numbers, and is applied to each sublist separately. This gives {7, 9, 11} and {3, 5, 13}.

How many comparisons and swaps does this need? **[2]**

(iii) How many further swaps would the original algorithm need to sort the revised list {7, 9, 11, 3, 5, 13} into increasing order? **[1]**

3 The network below represents a number of villages together with connecting roads. The numbers on the arcs represent distances (in miles).



(i) Use Dijkstra’s algorithm to find the shortest routes from A to each of the other villages.

Give these shortest routes and the corresponding distances. **[6]**

Traffic in the area travels at 30 mph. An accident delays all traffic passing through C by 20 minutes.

(ii) Describe how the network can be adapted and used to find the fastest journey time from A to F. **[2]**

Section B (48 marks)

4 Simon has a list of tasks which he has to complete before leaving his home to go on holiday. The table lists those tasks, their durations, and their immediate predecessors. The durations assume that only one person is working on each activity.

Task		Duration (minutes)	Immediate predecessor(s)
A	pack suitcases	30	–
B	make up beds	10	–
C	clean upstairs	20	A, B
D	wash upstairs floors	10	C
E	bring in outside furniture	15	–
F	close down central heating	5	–
G	disconnect TV system	5	–
H	load car	10	A
I	clean downstairs	25	D, F
J	wash downstairs floors	10	I
K	wash patios	15	E
L	lock up	5	G, H, J, K

(i) Draw an activity on arc network for these activities. **[5]**

(ii) Mark on your diagram the early time and the late time for each event. Give the minimum completion time and the critical activities. **[6]**

(iii) Explain why Simon will require help if the tasks are all to be completed within the minimum time. **[1]**

Simon's friend offers to help. They share the tasks between them so that each task is completed by only one person.

(iv) Produce a cascade chart to show how the tasks can be shared between Simon and his friend, and scheduled, so that the project can be completed in the minimum time. **[4]**

5 Angelo manages a winter sports shop in a ski resort. He needs to decide how many snowboards and how many pairs of skis to purchase for the coming season to maximise his profit from hiring them out.

He has space for at most 250 snowboards and 500 pairs of skis.

Because there are more skiers than snowboarders Angelo will purchase at least 10% more pairs of skis than snowboards.

Both snowboards and skis need servicing, and his servicing facility can cope with no more than 600 units (ie snowboards or pairs of skis).

His expected profit from buying and renting out a snowboard is €40 for the season, and his expected profit from buying and renting out a pair of skis is €50 for the season.

(i) Define appropriate variables, construct inequality constraints, and draw a graph representing the feasible region for Angelo's decision problem. **[10]**

(ii) Give the objective function and find the solution which will give the maximum profit. **[3]**

Angelo considers increasing the cost of snowboard hire so that snowboard profits rise enough to change the optimal solution.

(iii) By how much will snowboard profits have to rise to change the optimal solution? **[1]**

Angelo increases the cost of snowboard hire and creates extra storage space for snowboards.

(iv) What is the greatest number of extra snowboards it is worth Angelo accommodating? **[2]**

[Question 6 is printed overleaf.]

6 The time intervals between customers arriving at the queue for the till in a small supermarket are modelled by the following probability distribution.

Time interval (mins)	1	2	3	4
Probability	0.3	0.5	0.1	0.1

(i) Give a rule for using 1-digit random numbers to simulate inter-arrival times. **[2]**

(ii) Use the nine random digits provided to simulate nine inter-arrival times. Hence, assuming that the first customer arrives at the queue at time 0, give the arrival times of the first ten customers. **[3]**

Customers shop for single items, light loads, medium loads or heavy loads. These require respectively 0.1, 0.25, 1 and 2 minutes on average to process at the till. The proportions in each category are $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}$ and $\frac{1}{7}$ respectively.

(iii) Give an efficient rule for using 2-digit random numbers to simulate till processing times. **[3]**

(iv) Use the 2-digit random numbers provided to simulate the till processing times for the first ten customers. There are more random numbers provided than you will need. **[2]**

60% of customers pay by credit card and 40% pay by cash. A credit card transaction takes 1 minute on average, and a cash transaction takes 0.25 minutes.

(v) Give an efficient rule for using 1-digit random numbers to simulate payment times. **[1]**

(vi) Use the ten random digits provided to simulate the payment times for the first ten customers. **[1]**

(vii) Use your answers to parts **(ii)**, **(iv)** and **(vi)** to find the departure times for the first ten customers. **[2]**

The shop owner is considering installing a second till which does not have credit card facilities. All customers paying cash will use this till.

(viii) Repeat part **(vii)** under this proposed new arrangement. **[2]**

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